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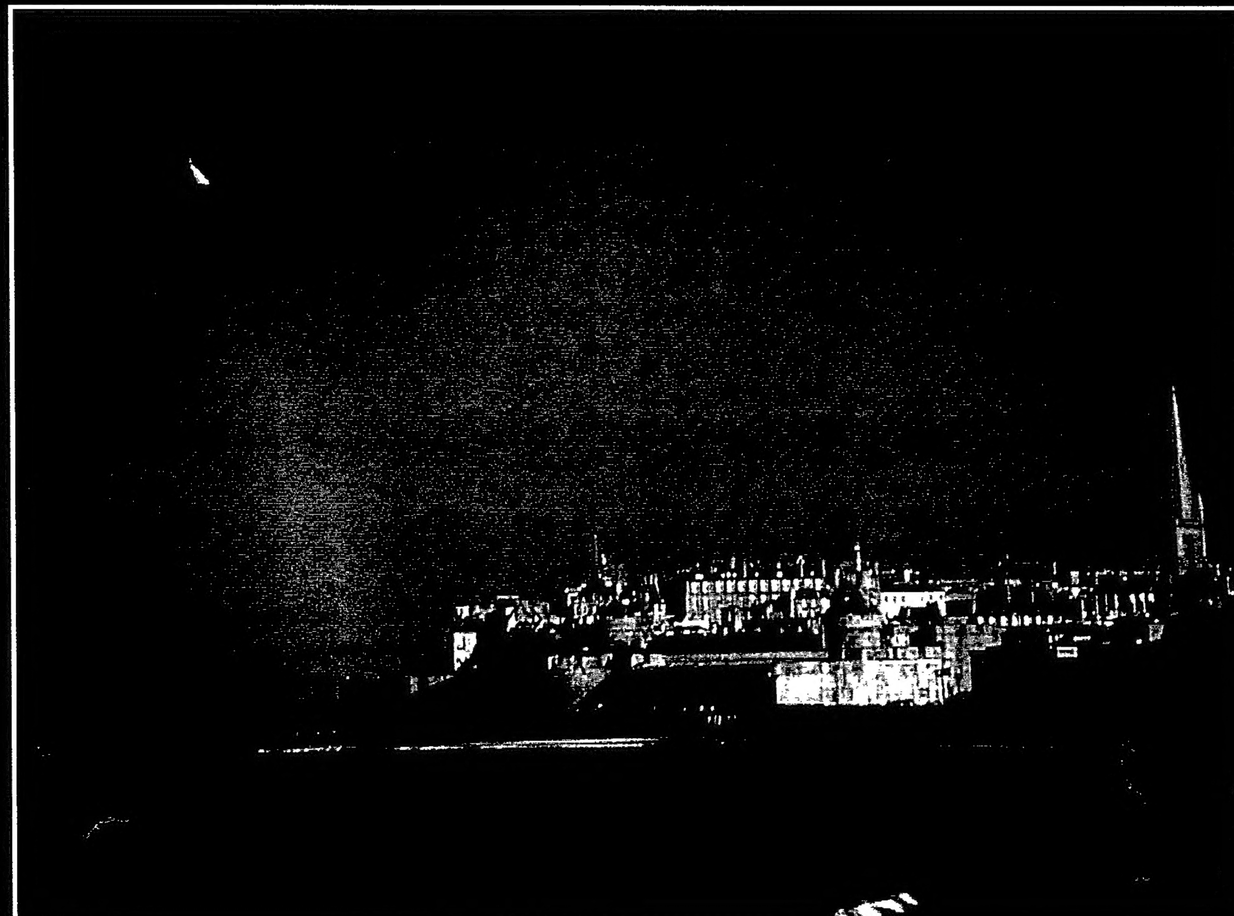

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Higher Order Sparse Grid Methods for Elliptic Partial Differential Equations with Variable Coefficients

S. Achatz*, C. Zenger

Keywords: *Sparse grids, hierarchical finite elements, higher order finite elements, numerical methods for PDEs.*

In our talk, we present a method for discretizing and solving general elliptic partial differential equations on sparse grids employing higher order finite elements. Due to the hierarchical structure of the function spaces, an elementwise assemblation of the system matrix is not feasible. Unidirectional algorithms are capable to compute the matrix vector multiplication for the Laplacian operator in $O(N)$ time, N the number of degrees of freedom. They fail when variable coefficients are introduced.

Therefore, we use an approach that evaluates the integrals approximatively. On the one hand, this approach is charactarized by its simplicity; the calculation of the occurring functionals is composed of basic pointwise and unidirectional algorithms providing $O(N)$ complexity. On the other hand, numerical experiments prove our method to be robust and accurate. Discontinuous coefficients can be treated as well as curvilinear bounded domains. When applied to adaptively refined sparse grids, our discretization results to be highly efficient, yielding balanced errors on the computational domain.

Applied to d – dimensional problems, traditional finite elements produce errors with respect to the energy norm of $O(N^{-p/d})$, sparse grids achieve an accuracy of $O(N^{-p})$ up to logarithmic factors. A comparison of our code to a comercial code based on traditional finite elements has shown clearly this substantial advantage of sparse grids.

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Techniques for Surface Reconstruction

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Keywords: *Surface Reconstruction, Delaunay Triangulation, Voronoi Diagram, Gabriel Graph, Natural Neighbours, Normal Estimation.*

Surface Reconstruction is a problem that deals with the polygonal approximation T of a surface M, sampled with a set of points S, which are either interpolated or approximated by T. We are going to deal only with cases when T interpolates S. T is usually chosen to be a triangular approximation of S, usually a subset of the Delaunay Triangulation of S. This means that first a Delaunay Triangulation of S is calculated and then an algorithm is used to chop off the triangles that do not belong to the approximation of M. This methodology is constantly gaining ground despite the heavy criticism it receives. This criticism is based on the bottleneck that the Delaunay Triangulation imposes on the complexity of the algorithm that implements this methodology. Recent parallel and serial implementations of the Delaunay Triangulation are very fast making this kind of criticism obsolete. We are going to present in a uniform way several algorithms proposed in the literature implementing the above methodology. Starting with Amentas and Deys algorithm, passing through Boissonnats we are going to end up in an algorithm that is based on the work of Adamy, i.e we are going to use the Gabriel Graph of S in order to get T.

One of the serious drawbacks that many surface reconstruction algorithms have, is that the calculation of the normals is necessary. These normals can be calculated by a k-NN scheme combined with a PCA approach or by the assistance of the Voronoi diagram. A k-NN scheme implies a near uniform density. The use of the Voronoi diagram implies an increase of the memory required for the algorithm to be executed. The final algorithm we are going to present does not require the calculation of the normals. A lot of researchers insist that in order to get an oriented manifold the calculation of normals is necessary. We are going

to present a manifold extraction scheme that does not need this information to extract an oriented manifold. Some results can be seen in [1].

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Conic Sections within CAD Systems

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Keywords: *CAD systems, conic sections, invariants, foci.*

Conic sections, i.e., ellipses, hyperbolas and parabolas, have for a long time been very important curves in applications such as architecture, mechanical engineering, and the airplane industries. They have therefore been integrated in many modern CAD–systems; in Version 5 of Dassault Systèmes CAD–system CATIA, e.g., they may be constructed by means of their foci. On the other hand, due to their many advantages, the popularity of NURBS and rational Bézier representations of curves and surfaces has hugely increased in the past years.

Due to these two main representations of conic sections in use, i.e., the one based on the geometrical invariants such as foci, center, axes, vertices on the one hand, and the rational quadratic Bézier form on the other hand, an easy transition between them has to be guaranteed. Thus, this talk deals with the problem of determining the above geometrical invariants of a non–degenerate conic section from its rational quadratic parametrization. We present an easy and unified geometrical approach for the determination of the conic's foci from which the remaining invariants such as center, axes, vertices are derived. In contrast to the other existing approaches [1,2,3] the main idea of this approach is based on the projective definition of the foci of a non–degenerate conic section, and is likely to allow a straightforward generalization to quadric surfaces in three–dimensional space.

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A Scattered Data Approximation Scheme for the Multidimensional Poisson Equation by Cardinal Radial Basis Interpolants

G. Allasia

Keywords: *Laplace and Poisson equations, approximation scheme, scattered data, cardinal radial basis.*

For certain class of physical problems, it would be desiderable to solve partial differential equations by means of discrete models which do not require any structure of nodes and are not too sensitive to dimensionality. E. J. Kansa introduced the concept of solving hyperbolic, parabolic and elliptic equations using Hardy's multiquadrics. Then several authors extended the use of Hardy's multiquadrics on the numerical solution of various partial differential equations. Two important features of the method had been observed: it gives a truly mesh-free algorithm, its computational complexity does not increase consistently with the spatial dimension.

In this paper, we consider a seemingly new scheme for discretization, which recalls Kansa's ideas but differs in some noteworthy aspects. We will limit ourselves to the consideration of the Dirichlet problem for the Laplace and Poisson equations. Let $\Omega \subset \mathbb{R}^s$, $(s \geq 2)$, be an open, simply connected point set, bounded by a piecewise smooth hypersurface Σ, and let $h(x) \in C(\Omega \cup \Sigma)$ and $k(x) \in C(\Sigma)$ be given real functions with $x = (x_1, x_2, \ldots, x_s)$. The well-known Dirichlet problem for the Poisson equation is that of finding a real function $u(x)$ in the space $\mathcal{U} = \{u \in C^2(\Omega) \cap C(\Sigma)\}$ such that

$$\Delta u(x) = -h(x)\,, \quad \text{for} \quad x \in \Omega, \qquad u(x) = k(x)\,, \quad \text{for} \quad x \in \Sigma\,. \tag{1}$$

To build up the discrete problem associated with (1), we consider a set of distinct points $S_N = \{\xi_i,\ i = 1, \ldots, N\}$, in general arbitrarily distributed in the domain Ω, and a suitable family of cardinal basis functions $g_k \in C^2(\Omega \cup \Sigma)$, $(k = 1, \ldots, N)$, such that $g_k(\xi_i) = \delta_{ki}$, where δ_{ki} is the Kronecker delta. The set $\mathcal{F}_N = \text{span}\{g_k,\ k = 1, \ldots, N\}$ is a linear space of dimension N and a subset of $\mathcal{U}$. The discrete problem, associated to the continuous boundary value problem, consists in finding $F \in \mathcal{F}_N$ expressed in the form

$$F(x) = \sum_{k=1}^{N} c_k g_k(x)\,.$$

and satisfying the linear system

$$\begin{aligned} \sum_{k=1}^{N} c_k \Delta g_k(\xi_i) &= -h(\xi_i)\,, \quad \text{for} \quad \xi_i \in \Omega, \\ \sum_{k=1}^{N} c_k \Delta g_k(\xi_i) &= -k(\xi_i)\,, \quad \text{for} \quad \xi_i \in \Sigma. \end{aligned} \tag{2}$$

Note that, owing to the cardinality of g_k, the approximation $F(x)$ to $u(x)$ satisfies the relation

$$u(\xi_i) \approx F(\xi_i) = \sum_{k=1}^{N} c_k g_k(\xi_i) = c_i\,.$$

The system matrix in (2) has dimension N, the number of points of S_N, but simple manipulations permit to obtain an equivalent system whose dimension equals the number of nodes belonging only to Ω. This reduced system is symmetric and strictly diagonally dominant, so that a unique solution exists and can be calculated by Gaussian elimination method, which do not require pivoting and is stable with respect to rounding errors. The system matrix has, in general, an acceptable condition number and changes only a little with spatial dimension.

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Arrangement of Lines in the Euclidean Plane: Representation and Topology

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Keywords: *Computational geometry, lines arrangements, isotopy, representation criteria.*

Lines arrangements are fundamental data structures for many computational geometry problems [1,3], and for some problems in interpolation theory [2]. Indeed, they bind to deal with both combinatorial and geometric information, in a coherent way.

The combinatorial issues are more easily solved in the projective plane, but for representation issues, the natural space is the euclidean plane, where the actual design has to be clear and legible. Given an arrangement, i.e. respecting its initial topology, we aim to avoid almost-degenerated representations, where some edges are too short, or two lines have too near slopes. We will give several numerical criteria, allowing to characterize the representation "niceness" of an euclidean lines arrangement, and some results on the existence of optima for these criteria. We then will examine some theoretical and algorithmical aspects of arrangements re-design.

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Approximating an Implicit Curve or Surface

D. Amar*, B. Mourrain, M. Yvinec

Keywords: *Implicit Curve, Topology, Approximation, Bernstein basis, Delaunay triangulation.*

The aim of this work is to build a piecewise linear approximation of an implicit curve or surface, preserving the topology. Implicit objects are defined as the zero level set of a function. Different methods exist to represent them (grid, marching cube, sampling,...) but they usually require a lot of computations and do not guaranty the topology.

We propose a new algorithm with guarantees on the topology and such that the number of linear pieces of the approximation is sensitive to the geometry of the implicit object. More precisely our algorithm applies to objects defined by polynomial equations or spline equations (A-patches) and ensures that the topology of the implicit curve (or surface) is catched within a precision ϵ, where ϵ is a tunable parameter. This means that the topology of the approximation is the same as the topology of the implicit object except for a few boxes of size ϵ on which different scenarii are proposed.

We use Bernstein bases to represent the function on a domain subdivided by axis parallell hyperrectangles or by a Delaunay triangulation. Using Descarte rules, the control coefficients give us an upper bound on the number of intersections between the implicit object and the edges of a cell and also the parity of this number of intersections. Subdividing the cells using the de Casteljau algorithm (according to one of the axis for the hyperrectangles and through Delaunay refinement for triangles), we obtain the control matrix in the new subcells . We

subdivide until the problem in each cell boils down to a trivial cases in which either the size of the cell is smaller than ϵ or the implicit object is proven to be homotopic to its linear approximation in the cell.

The advantage of the Delaunay triangulation is that it implies no choice of specific directions. The number of linear pieces of the approximation depends on the number of critical points, on a "local feature size" and on the parameter ϵ. Experiments will be presented to compare the different representations and subdivision strategies.

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A General Multiresolution Method for Fitting Functions on the Sphere

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Keywords: *Spline wavelets, multiresolution analysis, compression of a closed surface.*

Recently, T. Lyche and L.L. Schumaker have described a method for fitting functions of class C^1 on the sphere which is based on tensor products of quadratic polynomial splines and trigonometric splines of order three associated with uniform knots. In this paper, we present a multiresolution method leading to C^2-functions on the sphere, using tensor products of polynomial and trigonometric splines of odd order with arbitrary simple knot sequences. We determine the decomposition and reconstruction matrices corresponding to the polynomial and trigonometric spline spaces. We describe the general tensor product decomposition and reconstruction algorithms in matrix form which are convenient for the compression of surfaces. We give the different steps of the computer implementation of these algorithms and finally we present a test example.

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Reproducing Kernels and Differential Riccati Equations

L. Amodei

Keywords: *Reproducing kernel Hilbert space, linear quadratic regulator problem, differential Riccati equation, algebraic Riccati equation.*

In the classical linear quadratic regulator (LQR) problem, the feedback solution is obtained from a symmetric positive operator $P(t)$ which is the solution of a differential Riccati equation. By using a general Hilbert space setting, we show that the operator $P(t)$ is the restriction to the diagonal $t = s$ of a reproducing kernel $P(t, s)$ associated with a reproducing kernel Hilbert space (RKHS). For a given evolution differential operator, it is possible to consider a pair of dual RKHS spaces and their respective reproducing kernels are the solutions of two dual Riccati equations.

We also discuss the case of the algebraic Riccati equation which is introduced by the infinite horizon LQR problem. In this case, the reproducing kernel is a function of $(t-s)$, the diagonal operator is thus constant and verifies an algebraic Riccati equation.

Finally, we give some examples for finite and infinite dimensional operators.

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A Multiresolution Analysis using Polyharmonic Splines

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Keywords: *Polyharmonic splines, wavelets.*

Multiresolution analysis and pre-wavelet decomposition of $L^2(\mathbb{R}^d)$ based on polyharmonic B-splines is developed here. Filters are computed and formulas for the analysis-synthesis of the signal are given. The connected algorithm is then implemented in the two dimensional case and some numerical examples and error estimates are presented.

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Mollification Formulas and Implicit Smoothing

R.K. Beatson

Keywords: *Radial basis function, data smoothing.*

This paper concerns fitting a smooth function to noisy data. It starts from the elementary observation that a radial basis function

$$s = p + \sum_{i=1}^{N} d_i \Phi\left(\cdot - x_i\right),$$

defined on R^n, can be smoothed by convolution against a smooth kernel k of sufficiently fast decay yielding the smoother spline

$$s = q + \sum_{i=1}^{N} d_i \Psi\left(\cdot - x_i\right),$$

where $\Psi = \Phi * k$ and $q = p * k$. In particular if one does things backwards and specifies that Φ is a power of the mod, or 2-norm, $\Phi(\cdot) = |\cdot|^{(2k-1)/2}$ and Ψ is the generalised multiquadric basic function $\Psi(\cdot) = \left(\cdot^2 + c^2\right)^{(2k-1)/2}$, then the corresponding convolution kernel k turns out to also be a generalised multiquadric, but with a negative index.

Computationally one proceeds by making an interpolatory fit to the noisy data using a radial basis function based on sums of shifts of Φ. Then the convolution is simply carried out implicitly by keeping the same RBF coefficients but replacing Φ with Ψ when evaluating the RBF. Note that no explicit convolution, or passing to the Fourier domain is required. Thus the method avoids the use of an FFT and can be applied when the data is scattered rather than gridded. Fortunately in many cases both the initial unsmoothed fit and the smoothed evaluation can be carried out with fast methods.

Examples will be given showing the utility of this method for smoothing 2D data, and also for smoothing implicit surfaces corresponding to point clouds in 3D.

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Semi-Cardinal Interpolation for Multivariable Splines

A. Bejancu

Keywords: *Semi-cardinal interpolation, multivariable splines, Wiener-Hopf factorizations.*

A semi-cardinal grid consists of all multi-integers whose last coordinate, say, is nonnegative. The talk will discuss the recent construction of multivariable spline interpolation schemes on semi-cardinal grids, outlining results, applications and open problems of this new research topic.

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Taylor Series Methods for Curvature Estimation and Curvature Extrema Detection

Alexander Belyaev

Keywords: *Taylor series, curvature estimation, curvature extrema detection.*

The paper consists of two parts (studies). Taylor series expansions constitute the main mathematical tool in both studies.

The first part is devoted to studying asymptotic properties of three-point approximations of the curvature of a smooth curve. In particular, it is shown that certain three-point curvature approximations are the best for such famous and classical curves as the Cornu spirals (Euler spirals, clothoids) and elastica curves. General asymptotic properties of three-point curvature approximations unaffected by the rigid motions are also studied.

The second part deals with smooth surfaces: analytical descriptions of the maxima and minima of the principal curvatures along their corresponding curvature lines are obtained for surfaces given in explicit and implicit forms. Since such curvature extrema involve 4th order surface derivatives, their analytical description is a challenging task, especially if a surface is given in implicit form. The obtained formulas lead to mathematically correct methods for detection of perceptually salient curvature extrema on implicit surfaces and range images. Numerical experiments demonstrate advantages of the developed mathematical methods.

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Reverse Engineering from Noisy Data of Objects Defined by Algebraic Surface Patches

M. Luzon, E. Pavlov, M. Bercovier*

Keywords: *Reverse engineering, algebraic surface patches , noisy data.*

In a previous work the authors solved the case of objects made of (large) collections of planar patches. Here the case of an object defined by a collection of k quadric surface patches is studied . The actual input data are n points in $3D$, sampled from the original patches with uniformly distributed random errors. An algorithm which reconstructs surfaces made of algebraic surface restricted patches of quadric order from the cloud of points, in $O(k^{2*9})$ is introduced. The complexity depends on the degree of the surface patches and on their number . While this could be defined as a general clustering (classification) problem , better algorithms can be devised by using the geometric and algebraic properties of the patches.

The reconstruction is performed by defining a distance and using hyper planes clustering in multidimensional spaces (the dimension being defined by the algebraic surfaces defining implicit equation). The proximity of the resulting surfaces can be expressed in terms of both the distances used and the surfaces coefficients. The algorithm is relatively robust. We provide theoretical and experimental evidence for the algorithm success and the possible pitfalls.

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On Extremal Problems of Navigation and Approximation of Surfaces

V.I. Berdyshev

Keywords: *Approximation, navigation, informativity, best trajectory.*

In the problem of navigation with reference to a geophysical field (earth surface) and its fragment connected with given vision set an informativity property of field considered. The notion of the informativeness modulus of a field is introduced. The approximation ensuring the best navigation is discussed. The navigation error caused by replacing the geophysical field with an approximating function p from a prescribed linear class $\mathcal{P}$ is investigated. Directional differentiability conditions are obtained for navigation error as a function of $p \in \mathcal{P}$. These results are used to derive necessary conditions for the approximating function $p \in \mathcal{P}$ that ensures a minimum of navigation error. Necessary conditions are given for smooth trajectory to minimize the derivative of the informativeness modulus (along a trajectory) at zero. Extremal problem of constructing of the best vision set is also considered.

The work was supported by the Russian Foundation for Basic Research, project no.02-01-00782.

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Scattered Data Wavelet Interpolation

C. Bernard*, S. Mallat, J-J. Slotine

Keywords: *Interpolating wavelets, scattered data, radial basis functions.*

We describe in this contribution an adaptive wavelet interpolation scheme designed for scattered sample interpolation. The interpolant is built iteratively as a combination of interpolating wavelets selected from a uniform square grid wavelet basis.

Asymptotic complexity and accuracy are analysed in a generic framework. The adaptivity of the scheme to sample and signal irregularity is also analysed.

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Recursively Generated Graph Surfaces

Martin Bertram*, Hans Hagen

Keywords: *Functional data, multiresolution methods, subdivision surfaces, terrain modeling.*

Subdivision surfaces are widely used for representing and rendering surfaces of arbitrary topology and they provide an ideal tool for multiresolution modeling of complex geometry [2]. Since subdivision surfaces are designed for representing manifolds embedded in three-dimensional space, they have scarcely been used for modeling graph surfaces $z = f(x,y)$. The problem with most subdivision schemes is that all three coordinates of the defining mesh need to be treated by the same subdivision rules to produce a smooth limit surface. In the case of graph surfaces, however, the smooth subdivision process applies to only one coordinate, e.g. the z-component, whereas linear subdivision is used for the coordinates of the domain, e.g. x and y.

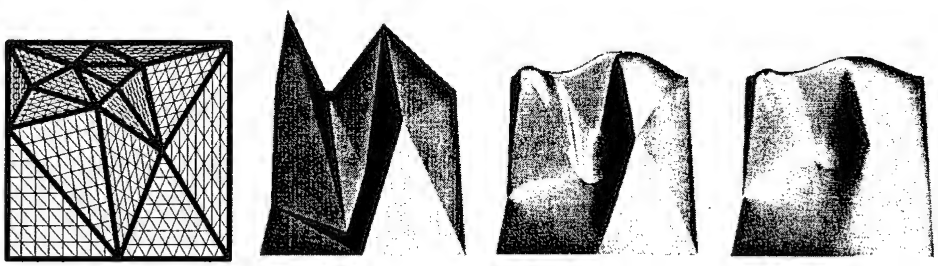

Fig. 1. a) Triangulated domain; b) base mesh; c) Loop subdivision applied to ordinates; d) our modified subdivision for graph surfaces.

We propose a modification of Loop subdivision surfaces [1] for the recursive generation of graph surfaces, such that the planar domain is not warped by the subdivision process in order to avoid creases, see figure 1. Starting with a triangulated planar domain associated with scalar values at the vertices, our scheme applies linear, stationary subdivision rules, recursively refining this mesh. The resulting sequence of triangulated meshes represents a smooth graph surface at multiple levels of approximation. During the subdivision process, it is feasible to add further geometric detail to a surface by displacing the ordinates associated with mesh vertices. This geometric detail can be expanded, for example, from sparse sets of wavelet coefficients.

The novelty of our scheme is that it applies subdivision only to the ordinates associated with vertices of the refined mesh, whereas the triangulated domain is linearly subdivided. Our subdivision scheme thus defines a sequence of nested spaces spanned by locally-supported bivariate basis functions, which can be used for interpolation or approximation (based on least-squares fitting) of prescribed functional data. Our approach is nearly as efficient as the original Loop scheme and well suited for the rendering of large datasets.

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Adaptive Tree Approximation for Progressive Compression of Surfaces

Peter G. Binev

Keywords: *Near-best tree approximation, triangulations, adaptive refinement, newest vertex bisection, progressive transmission, mesh simplification, multiresolution, subdivision surfaces.*

Dense triangular meshes are one of the most common representations of highly detailed surfaces in computer graphics. These complex meshes require a huge storage space and are difficult to handle even in common operations such as rendering, editing and transmission. In the last decade multiresolution representations have been established as the key for these problems. There are different approaches but all of them reduce the given dense triangulation τ to a triangulation τ_0 of low complexity. Most of the procedures require remeshing after each step in order to receive a proper triangulation.

In this paper we consider triangulations τ received from τ_0 by several applications of so-called *newest vertex bisection*. This procedure can be viewed as growing a binary tree from each triangle from τ_0 in order to receive the *master tree* corresponding to τ. Then, the simplification of the mesh corresponds to trimming some brunches of the master tree. This model is often used in Adaptive Finite Element Method and it does not require remeshing.

The tree structure gives the possibility to obtain progressive compressions of the given surface by applying an algorithm with linear complexity that finds consecutive near-best approximations.

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Optimizing Basis Functions For Best Approximation

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Keywords: *Approximation, interpolation, order of approximation, asymptotic approximation constant, spline, wavelet, integer-shift-invariance.*

By evaluating approximation theoretic quantities we show how to compute explicitely the basis generators that minimize the approximation error for a full set of functions to approximate. We give several examples of this optimization, either to get the best generators that have maximal order for minimum support[1], or to design the best interpolation scheme with classical generators, such as B-splines[2]. We present practical examples that visually confirm the validity of our approach.

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Adaptive Bivariate Interpolation by Multiquadrics Perturbed in Scale and Shape

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Keywords: *Positive definite basis functions, shape parameters, scaling.*

For knots scattered in a two–dimensional domain, we present a basis that consists of finite differences of multiquadrics with a common scale parameter. For such a basis the matrix of interpolation is nonsingular. Furthermore, there is a suitable adaptive choice of the scale parameter, in relation to what is suggested in the literature [1-3]. This basis has been validated numerically on samples of size not large with the purpose of providing surfaces with pleasant graphical behavior. Special examples for functions with sigmoidal or asymmetric radial structure are provided.

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Fast Approximation for Tensor Product Structures with Applications for Blending

K.-H. Brakhage

Keywords: *B-spline, approximation, blending, reparametrization.*

Most CAGD systems are based on parametric curves and surfaces, e.g. Bézier, B-spline or Nurbs surfaces. Creating complex objects advanced features such as surface trimming and blending are needed. Unfortunately the resulting surfaces are no longer of the original kind. In practice this often leads to gaps and the patches are far away from smoothly joining. This paper introduces some new techniques for repartitioning and reparametrization of surfaces to avoid the above deficiencies. The methods are based on fast approximation and interpolation schemes for tensor product structures. The theory behind that is based on a norm equivalence for matrices of Kronecker product type. Additionally,

global fairing is provided. In a first step we partition the parameter space of the surfaces and generate regular quadrilateral grids inside the new partitions. Well-known techniques for grid generation can be used here, e.g. [1]. The grids determine the new structure of the patches. Then we compute some data points with the original surface maps. These points are used to compute the new patches using our fast techniques for approximation and/or interpolation. Fairness criterions can optionally be included. We use the methods for surface blending and preprocessing of surfaces for grid generation. All methods are not restricted to surfaces. They can be extended to parametric spline volumes.

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Automatic Construction of B-spline Surfaces from Adaptively Sampled Distance Fields

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Keywords: *Distance fields, geometric modeling, surface fitting, surface reconstruction, B-spline surfaces.*

We present a method for automatically constructing B-spline surfaces from models described by adaptively sampled distance fields (ADFs [1]). Our approach begins by constructing an octree representation of the ADF. Leaves of the octree represent the zero set of the distance field and a voxel approximation of the model. The outside faces of the leaves are then used to construct a quadrilateral mesh. Through a series of Laplacian edge-based smoothing operations and ADF-directed projection steps, we obtain a mesh that more accurately represents the model.

The resulting mesh contains many extraordinary (non-valence-four) vertices. Due to the structure of the mesh induced by the octree, we can apply a mesh simplification procedure that greatly reduces the number of extraordinary vertices. Simplification consists of collapsing single quadrilaterals or entire "rings" of quadrilateral strips. To mitigate the effect of mesh distortion and to help separate extraordinary vertices during this simplification stage, we apply several subdivision steps and several smoothing and projection steps.

Quadrilateral B-spline patch boundaries are extracted from the simplified mesh. These boundaries are created by finding paths between the remaining extraordinary vertices. The quadrilateral regions are then sampled to generate bi-cubic B-spline patches. Patches are C^2 continuous on the interior and are C^0 continuous along patch boundaries. A variant of an algorithm presented by [2] is then applied to ensure G^1 continuity between patches.

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Adaptive Numerical Integration Using Sparse Grids

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Keywords: *Multivariate numerical quadrature, adaptive grids, sparse grids, higher order approximation.*

In this contribution, we study the potential of adaptive sparse grids for multivariate numerical quadrature in the higher dimensional case, i. e. for a number of dimensions beyond three and up to several hundreds. There, conventional methods typically suffer from the curse of dimension or are unsatisfactory with respect to accuracy. Our sparse grid approach, based upon Smolyak's tensor product decomposition, overcomes this dilemma to some extent, and introduces an additional flexibility with respect to both the order of the 1D quadrature rule applied and the placement of grid points. The presented algorithm is applied to some test problems and compared with other existing methods.

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On Chung and Yao's Geometric Characterization for Bivariate Polynomial Interpolation

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Keywords: *Bivariate polynomial interpolation, geometric characterization.*

Chung and Yao's geometric characterization for sets of points in the plane gives rise to simple Lagrange interpolation formulae in the space of polynomials of degree not greater than n. A conjecture on these sets states that *there exists a line containing* $n+1$ *nodes*. This conjecture has only been proved for degrees ≤ 4. In this paper, we analyze some consequences of assuming the validity conjecture for arbitrary degree, which provide a better knowledge of Chung and Yao's characterization. Their geometric characterization can be easily checked in each case but it is very difficult to describe all configurations satisfying it. One of the main results of this paper is the equivalence of the above mentioned conjecture with the existence of at least 3 lines containing $n+1$ nodes.

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Adaptive Data Fitting Based on Wavelets

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Keywords: *Curve and surface fitting, wavelets, adaptive methods, sparse data, robust regression.*

Given a set of irregularly distributed data containing outliers, the aim is to fit a curve or surface to the data, and to determine an efficient representation. Particular emphasis is placed on the fact that the data may be very sparse and irregularly spaced. Therefore, we search for an adaptive representation without introducing a high complexity everywhere from an underlying virtual fine grid.

To achieve this goal, we employ B–Spline–Wavelets within a Least–Squares–Approach for data fitting. In an adaptively constructed grid, starting on some coarse level, wavelets from finer levels are successively taken into account according to some criterion depending on the distribution of the data. In this way, we try to balance between achieving a sufficiently good representation and artefacts from overfitting. After computing the wavelet coefficients, a thresholding procedure on each level guarantees that only relevant information is processed. Furthermore, the update–character of the resulting adaptive wavelet representation allows for efficient local changes.

In the application, we focus on the problem of robust regression, that is, fitting data which is known to a certain degree to be obtained from inaccurate or wrong measurements by taking into account local statistical properties of the data.

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Decomposition of the Inverse Fourier Transform of 1-radial Functions and Derivatives

W. zu Castell

Keywords: *Radial functions, fractional derivatives, Dirichlet splines, Meijer's G-function.*

H. Berens, Y. Xu and the author proved that the inverse Fourier integral of 1-radial functions, i.e., functions which are radial w.r.t. the ℓ_1-norm in $\mathbb{R}^d$, can be decomposed into a multi-dimensional integral with B-spline kernel and a transformation on $\mathbb{R}_+$ with a kernel given by a Meijer's G-function. The latter can be seen as an analogue of the well-known Fourier-Bessel transform.

Here we associate to the underlying transform weights. There is an analogue development where the B-spline has to be replaced by a Dirichlet-spline, while on $\mathbb{R}_+$ the related transform can again be defined via G-function kernels. This modification can be interpreted as fractional derivative of the inverse Fourier integral of 1-radial functions.

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On the Angular Defect of Triangulations and the Pointwise Approximation of Curvatures

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Keywords: *Surfaces, meshes, curvatures, approximations, differential geometry.*

Let S be a smooth surface of E^3, p a point on S, k_m, k_M, k_G and k_H the maximum, minimum, Gauss and mean curvatures of S at p. Consider a set $\{p_ipp_{i+1}\}_{i=1,\ldots,n}$ of n Euclidean triangles forming a piecewise linear approximation of S around p —with $p_{n+1} = p_1$. For each triangle, let γ_i be the angle $\angle p_ipp_{i+1}$, and let the angular defect at p be $2\pi - \sum_i \gamma_i$.

We first show that asymptotically —when the distances $\| pp_i \|$ go to zero, the angular defect does not provide, in general, an estimate for the Gauss curvature of S at p. More precisely, we show that the angular defect estimates k_G within an error bound depending upon k_m and k_M.

If it is assumed that the normal to S at p is known and that the p_is lie in normal sections two consecutive of which form an angle of $2\pi/n$, we provide the closed form expression of the angular defect as a function of the principal curvatures and $2\pi/n$. In particular, we show that $n = 4$ is the only value of n such that $2\pi - \sum_i \gamma_i$ depends upon the principal directions, and $n = 6$ is the only value such that $2\pi - \sum_i \gamma_i$ provides an exact estimate for k_G. A corollary of these results is that the principal curvatures —whence the mean curvature k_H— can be computed from the angular defects of any two triangulations whose valences are not four. If in addition the angular defect of the valence four triangulation is available, one can compute the principal directions.

Since meshes are so ubiquitous in Computer Graphics and Computer Aided Design, and Discrete Differential operators are so ubiquitous for their processing, we believe these contributions are one step forward the intelligence of the geometry of meshes, whence one step forward more robust algorithms.

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Surface Fitting Validation Using Regression Analysis

T. Chaperon

Keywords: *Surface fitting, goodness of fit, residuals, regression, F distribution test, geometric primitives.*

Surface fitting is a crucial task in reverse engineering and 'As Built CAD Modeling'. In these applications, most surfaces are simple geometric primitives, e.g. the plane, the sphere, the cylinder, the cone or the torus. Such a surface model can be fitted to a set of 3D points using least-squares techniques [6,7]. Once this surface model has been fitted to the set of points, an important question is to determine whether it suitably fits the data or not. Assuming the noise is Gaussian with known variance, a simple χ^2 test may give an estimate of the probability that the model fits the data. However, a range scanner usually produces 3D data with some noise of unknown variance. Therefore, we propose a method for validating or invalidating a surface model without knowing the noise level.

The principle is to examine the distribution of the residuals over the surface. Examining the residuals as a goodness-of-fit test has already been investigated in [4,5] and [1]. In both works, a boolean value is associated with each point depending on the sign of the corresponding residual. This has yielded goodness-of-fit criteria for 2D curves [4,5] (using runs tests on boolean sequences) and for surface patches of the type $z = f(x, y)$ in a range image [1] (using erosion on a 2D grid). It is not straightforward to extend these concepts to the full 3D case. In contrast, we propose to examine the residuals directly, through a regression analysis. In other words, the residuals are approximated as a polynomial function of the two coordinates of each point on the surface. In the case of Gaussian residuals, which turns out to be a reasonable assumption in practice, this leads to a simple criterion based on an F-distribution test. Moreover, this criterion might be directly applied to any surface for which a parameterization is available.

Experiments are shown on the above mentioned primitives using both synthetic and real data sets.

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Kernel and Wavelet RBFs Based on Fundamental and General Solutions of Partial Differential Equations

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Keywords: *Radial basis function, wavelets, kernel RBF, general solution, fundamental solution, partial differential equation.*

The origin of the traditional RBFs (except the thin plate spline (TPS)) has little to do with partial differential equations (PDEs). Based on the second Green identity, the present author presented the kernel RBF via fundamental and general solutions of differential operators. The popular TPS and multiquadratics (MQ) are found to be special cases of the kernel RBF. Five types of the global and compactly-supported kernel RBFs were proposed. For example, e^{-ar} is much better than Gaussian, TPS or MQ for diffusion and convection-diffusion problems due to its underlying approximation to their fundamental solutions. The solvability of the RBF approximation of PDEs could also be proved based on the corresponding boundary integral equation theory.

The shape parameter in the MQ was understood to be an intelligent instrument simplifying complete fundamental solution. Thus, we could construct various operator-dependent pre-wavelets by simply replacing Euclidean distance variable of corresponding fundamental and general solutions with the MQ.

Unlike the most traditional expansion systems such as Fourier ones, the basis functions of the wavelet analysis, however, are normally not solutions of differential equation. By using the fundamental and general solutions of PDEs, we derived scale-orthogonal RBF wavelet series and transform. For instance, the harmonic wavelet RBF expansion of a real-value n-dimensional function f(x) is given by

$$f(x) = \alpha_0/2 + \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} \alpha_{jk}\phi_n(\lambda_j \|x - x_k\|)$$

where ϕ_n represents nonsingular general solutions of n-dimension Helmholtz equation (e.g. the zero order Bessel function of the first kind for 2D case, Sinc function for 3D case); and λ_j as nonuniform scale parameters are the zeros of ϕ. The above wavelet series is a natural extension of Fourier series with the features that the basis function has closed form expression, is dimension-dependent, keeps all strengths of the RBF, and tends to zero at infinity. The purpose of this paper is to summarize these results and report some latest advances based on them. For a complete reference see my homepage: http://www.ifi.uio.no/ wenc/html/rbf.htm.

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Quasi-interpolation Using Radial Basis Functions for Poisson Problems

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Keywords: *Radial basis functions, quasi-interpolation, Poisson's equation.*

In the past, quasi-interpolation schemes have been applied to avoid the difficulty of ill-conditioning problem in the literature of radial basis functions (RBFs). Recently, based on the theoretical results of Li and Miccelli [1] in the quasi-interpolation using RBFs, a class of new RBFs has been constructed and tested by Li, Ho and Chen [2]. Two attractive features of this new class of RBFs are (i) its flexibility of constructing RBFs (ii) the difficulty of ill-conditioning problem using traditional RBFs can be avoided since no matrix inversion is required in the process of approximation. The new development has opened the opportunity in the area of partial differential equations.

In this talk, we extend the newly established theoretical and numerical results mentioned above to solve Poisson problems. We first apply the method of particular solution to split the given boundary value problem into evaluating particular solutions and solving the related homogeneous equations. Coupled with the newly constructed radial basis functions, the dual reciprocity method (DRM) provides an efficient and accurate way to evaluate the particular solutions without domain/boundary discretization and integration. The method of fundamental solutions (MFS), a meshless and spectrally accurate boundary method, is adopted as the boundary method to solve the corresponding homogeneous equation. The same procedure can be extended to a broad type of partial differential equations. More significantly, the convergence result of the DRM using RBFs (interpolation and quasi-interpolation) for Poisson's equation has been further proved by Li and Golberg [3]. As a result, the numerical algorithm in our proposed approach is theoretically justified and the numerical results are robust.

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Discrete Geometrical Tools for CAGD Problems

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Keywords: *Algorithms, CAGD, discrete topology, exclusion, subdivision, voxels.*

We recall how subdivision-exclusion method works to localize geometrical objects and produce a set of voxels. Results from digital geometry are given and we show why local criteria for thinning can maintain global topological properties.

Neutral points are defined and we deduce algorithms to localize 2D or 3D objects which are defined by a set of equations. We prove the convergence of the algorithm. Several examples and applications are given.

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Monotone-Visibility: a Non-differentiable Generalization of Semi-convexity for Planar Shapes

Sung Woo Choi

Keywords: *Minkowski sum, semi-convex, convex, monotone-visibility, generalized convexity.*

A planar shape is *semi-convex* if the normal vector field along the shape boundary does not rotate concavely more than π. The semi-convexity for planar shapes, recently introduced in [1], is a generalization of convexity, which is preserved under the Minkowski sum. It is significantly more general than the usual convexity, and, in fact, more general than various existing notions of generalized convexity such as the star-shapedness. Also, the class of semi-convex domains is maximal among the classes of domains which are closed under the Minkowski sum. These results not only reveal the deep relationship between the topological and the geometrical properties of the planar shapes, but also are expected to open new and efficient computational methods for Minkowski sum.

In this paper, we introduce a new notion of visibility for planar shapes, called *monotone-visibility*. A planar shape is monotone-visible, if an observer outside the shape can see every point on its boundary by traversing around it with monotone rotation of the viewing direction. Equivalently, it means that the shape can be viewed from the infinity with monotone rotation of the viewing direction, if we regard the shape as lying in the projective plane.

We show that, in fact, the monotone-visibility is equivalent to the notion of semi-convexity, thus providing an interesting link between Minkowski sum and visibility. Furthermore, we show that the monotone-visibility can also generalize the semi-convexity even to the domains whose boundaries are not piecewise differentiable, whereas the current definition of the semi-convexity in [1] requires piecewise differentiability of the shape boundary since it needs the existence of the normal vector field along the boundary. In view of the fact that the semi-convextiy is a most natural generalization of the convexity in many respects, this removes a severe restriction of *a priori* differentiability for the semi-convexity.

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Edge-Adapted Nonlinear Multiresolution Representations

A. Cohen*, B. Matei

Keywords: *Wavelets, nonlinear approximation, anisotropy, edges, ENO interpolation.*

This talk will present a class of bidimentional multiresolution transforms which differ from standard wavelet decompositions in the sense that they are based on data dependent nonlinear operators. These operators are inspired from the ENO interpolation operators introduced by Harten and Osher in the context of numerical shock computations. The goal of the nonlinear processing is to perform within the tranform a specific treatment of edges which will take advantage of their geometric smoothness in order to obtain sparser representations and therefore better approximations. We shall address the new theoretical difficulties which occur when analyzing these methods, and present their practical performances on synthetic and real images.

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Univariate Monotone Smoothing of Noisy Discontinuous Data

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Keywords: *Smoothing splines, inf-convolution splines, monotonicity preservation.*

Given some noisy data coming from some process which is supposed to be monotone, continuous and with discontinuous first derivative at given points, we want to construct an approximation of the data which is preserving the features of the process.

The idea is to adapt both the results obtained by P.J. Laurent [1] on inf-convolution splines for approximating data by a function having discontinuous first derivative at some given points and the results obtained by S.N. Wood [2] on monotone smoothing approximation by cubic splines. We so build a monotone piecewise cubic function with prescribed discontinuities which smooths the data according to some suitable smoothing parameter.

To do so, assuming that $\{x_i, y_i\}_{i=1}^n$ are the data and $\{\hat{x}_j\}_{j=1}^m$ the position of the discontinuities ($n \gg m$), we minimize

$$E(f,\ell) = \int_{\mathbb{R}} (f"(x))^2 dx + \sum_{i=1}^n \rho_i \left(f(x_i) + \sum_{j=1}^m \ell_j (x_i - \hat{x}_j)_+ - y_i \right)^2$$

under the constraint $f + \sum_{j=1}^m \ell_j(\bullet - x_j)_+$ is a monotone function, with f belonging to the space of natural cubic splines g with knots $\{x_i\}_{i=1}^n$ such that $g''(\hat{x}_j) = 0$, $j = 1, \ldots, m$.

This is obtained by solving a quadratic programming problem with linear constraints based on sufficient conditions for monotonicity first used in [3] for monotonic interpolation with piecewise cubic functions.

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Geometric Construction of Spline Curves with Tension Properties

Paolo Costantini*, Carla Manni

Keywords: *Control polygon, tension properties, torsion continuity.*

The structure of polynomial splines curves, which constitute one of the most important tools in the interpolation or approximation of spatial data and in free form design, has been very soon enriched by *shape-parameters*, which allow modifications of their form. Typically, this is obtained at the price of a weakening in the mathematical structure, and the C^r smoothness of the curve is relaxed to geometric (GC) or Frenet continuity (FC). In this connection, it is important to point out that $C^2 - FC^3$ is a reasonable smoothness property, both from the mechanical point of view (the motion of a point on the curve has a continuous acceleration) and from the geometric point of view (the tangent and curvature vectors and the torsion are continuous). In many cases, these spline curves (with

or without shape-parameters) can be obtained using the so called *geometric construction* which, starting from the spline control polygon, produces the Bézier control points after some steps of a corner-cutting procedure.

In the applications of splines to data interpolation or approximation, the shape-parameters are used for obtaining a *shape-preserving curve*, that is a curve whose direction of the curvature and sign of the torsion are constrained to be the same as the data. Typically, the derivation of such constraints is very cumbersome but can be greatly simplified when the spline curves possess tension properties and are obtained with a geometric construction; in this case, roughly speaking, it suffices to construct a constrained spline control polygon and to suitably stretch the curve itself.

In this talk we propose a simple modification of the geometric construction for quintic C^4 polynomial spline curves and describe the structure and the properties of two classes of curves obtainable from it: quintic $C^2 - FC^3$ splines with tension properties and C^3, quintic like, variable degree polynomial splines. The applications of these spline (which do depend only on the spline control points and on the shape-parameters) in data interpolation and approximation will be discussed.

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Constrained Interpolation by Frenet Frame Continuous Quintics

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Keywords: *Shape-preserving, interpolation, parametric curves, geometric continuity.*

Shape-preserving interpolation via functional as well as parametric splines is a well studied topic for the planar case: an abundance of papers exists on this argument. On the other hand, introducing and validating a notion of shape-preserving interpolation for spaces curves is considerably more complex than for planar ones and the related literature is apparently limited.

On this concern, a considerable part of the available schemes only ensure geometric continuity of the obtained curve, [1], [5]. Recently, C^2 and C^3 shape-preserving interpolating space curves have been obtained using polynomial splines of variable degree, [2], [3]. However, working with low(fixed)-degree polynomial splines seems to be a standard choice in the CAD/CAM community. This motivate the careful investigation of shape preserving properties of classical cubic ν−splines recently carried out in [4].

Cubic ν−splines are $C^1 - GC^2$ curves but an higher analytic/geometric continuity is preferable in some applications. In this poster we discuss how the degrees of freedom of C^2 torsion continuous quintic splines can be exploited to obtain spatial shape-preserving interpolating curves. We consider both $C^2 - FC^3$ and $C^2 - FC^4$ quintics. For each case we present a global iterative algorithm for constructing interpolating space curves reproducing the shape of the polygonal line that interpolates the given data. The shape of the curve is controlled by the amplitudes of the tangent vectors at the data sites which play the role of tension parameters. The proposed algorithms are fully automatic, converge in a finite number of iterations and require at each iteration the solution of diagonally dominant linear system. For $C^2 - FC^3$ curves the scheme allows the flexibility of assigning tangent directions at data points while in the $C^2 - FC^4$ case the proposed approach presents some connections with the theory of τ−splines.

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Adaptive Wavelet Methods for Nonlinear Problems

A. Cohen, W. Dahmen*, R. DeVore

Keywords: *Tree approximation, best N-term approximation, nonlinear functions of wavelet expansion, adaptive application of operators, convergence rates, complexity estimates.*

This talk is concerned with the design and complexity analysis of adaptive wavelet methods for nonlinear variational problems. A key issues is the estimation and evaluation of wavelet coefficients of *nonlinear* functions of multiscale expansions. A central result is to show that certain sequence spaces, that can be used to measure the sparsity of tree type arrays of wavelet coefficients, are stable under a certain class of nonlinear mappings. We indicate how these results can be used to facilitate the sparse evaluation of wavelet coefficients of compositions at asymptotically optimal computational cost, and how this enters the adaptive

solution of nonlinear variational problems. The main idea is to transform the original problem into wavelet coordinates so as to arrive at a well posed problem in ℓ_2. The numerical realization of an ideal convergent iterative process for the latter problem relies then on the adaptive application of the involved operators to the current approximation within suitable dynamically updated accuracy tolerances. Again asymptotically optimal work/accuracy rates are obtained when compared with best N-term approximation.

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On Multivariate Nonlinear Approximation Spaces

S. Dekel

Keywords: *Nonlinear approximation, multivariate smoothness spaces, piecewise polynomials.*

It is known that classical smoothness spaces fail to characterize approximation spaces corresponding to multivariate piecewise polynomial nonlinear approximation. We formulate a measure of smoothness that combines measures of smoothness in several dimensions and show examples where this type of smoothness measure can almost characterize these spaces. This approach corresponds well to recent attempts to find a compact representation of multivariate signals, such as images, which outperform Wavelets. The question whether the characterization proposed in this work can be further 'simplified' remains open.

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Adaptive Thinning in Image Compression

Laurent Demaret*, Armin Iske

Keywords: *Thinning algorithms, scattered data interpolation, image compression, multiresolution methods.*

This talk concerns image compression using adaptive thinning ([1]). For this, a new customized encoding scheme is developed. This scheme is based on the transmission of scattered data points output by the adaptive thinning algorithm. Though adaptive thinning relies on Delaunay triangulations, the encoding scheme does not require to code the connectivity between the points. In order to improve the compression rate, the points are transmitted such that associated costs are minimized. This relies on a specific ordering of the points. Many efficient still image encoding schemes, e.g. those based on wavelets ([2,3]), make advantage of the correlation of the data. Likewise, our image compression scheme uses assumptions on the correlation. This allows us to reduce the data size, and thus leads to small encoding costs. The latter is also supported by using adaptive arithmetic encoding. Multiresolution aspects of the subject are addressed in this talk and, finally, selected examples show the performance of our method.

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Smooth Parameterization of Meshes and Applications

Mathieu Desbrun

Keywords: *Triangulations, parameterization, remeshing.*

As 3D data becomes more and more detailed, there is an increased need for fast and robust techniques to automatically compute least-distorted parameterizations of large meshes. In this talk, I will present new theoretical and practical results on the parameterization of triangulated surface patches. Given a few desirable properties such as rotation and translation invariance, I will show that

the only admissible parameterizations form a two-dimensional set and each parameterization in this set can be computed using a simple, sparse, linear system. Since these parameterizations minimize the distortion of different intrinsic measures of the original mesh, we call them Intrinsic Parameterizations. In addition to the theoretical analysis, I will propose robust, efficient and tunable tools to obtain least-distorted parameterizations automatically. In particular, I will give details on a novel, fast technique to provide an optimal mapping without fixing any boundary conditions, thus providing a unique Natural Intrinsic Parameterization. Other techniques based on this parameterization family, designed to ease the rapid design of parameterizations, will also be discussed. Finally, I will briefly describe a new interactive remeshing technique using these parameterizations.

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Fast Surface and Tree Structure Extraction of Vascular Objects in 3D Medical Images

T. Deschamps*, L.D. Cohen, S.M. Ebeid

Keywords: *Minimal path, segmentation, fast-marching algorithm, level-sets model, 3D medical imaging.*

We present a new fast approach for surface segmentation of thin structures, like vessels and vascular trees, based on Fast Marching and Level Sets methods. Malladi et al have demonstrated that the Fast-Marching algorithm can be used for surface segmentation. In this paper we show how it can be extended to long and thin objects, where the previous approach would usually fail. Once the surface of the tubular object is segmented, we can also extract a set of trajectories, and reconstruct the tree structure of this object (see figure 1)), to quantify the extent of pathologies like stenoses and aneurysms.

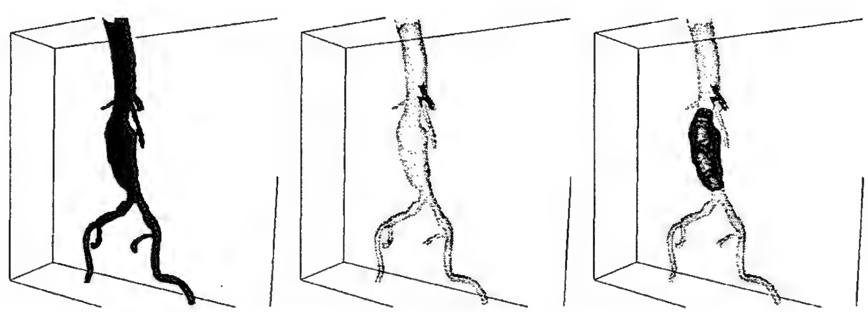

Fig. 1. An aorta segmented with its underlying tree structure.

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Ray Casting by Subdividing Algebraic Equations

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Keywords: *Ray casting, subdivision, localization, algebraic systems.*

We propose here to solve the problem of finding all rays that are emitted from a source toward an object and then reflected to an observer following geometric optics laws. The object is described by its boundaries using parametric surfaces (NURBS). We first give a set of equations that defines necessary conditions for a hit point to exist on a surface. Then we detail how to refine the problem (if necessary) in order to rewrite it locally.

This approach leads to a subdivision algorithm that converges to the solutions *i.e.* : that localizes on each surface, and at any desired precision, the parameter values (if any) that correspond to valid hit points. We show that this approach can be generalized to consider sources and observers that are not limited to just single points. This allows to support curves, surfaces or any "localizable" object as valid sources and/or observers.

Finally, we will show how this generalization is used to process multiple reflections on the geometric model.

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Encoding of Digitized Surfaces

R. A. DeVore

Keywords: *Digitized surfaces, data compression, classification of surfaces, optimal encoding.*

Digital Terrain Elevation Data (DTED) are data sets that give the altitude of various positions on the earth. They are used in many applications such as navigation and emergency preparedness. These data sets are huge and need to be stored, querried and displayed rapidly. This talk will discuss possible ways of compressing these data sets so as to meet the application goals. We are interested more in the development of a mathematical framework for DTED compression than in algorithmic development per se. Thus the talk will discuss such issues as apropriate metrics to measure error, classification of surfaces, and optimal encoding.

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Characteristics of Dual $\sqrt{3}$ Subdivision Schemes

N.A. Dodgson*, I.P. Ivrissimtzis, M.A. Sabin

Keywords: *Subdivision, sqrt-3, primal, dual.*

Primal $\sqrt{3}$ subdivision schemes were introduced by Kobbelt [2]. Kobbelt's scheme is *primal* in the sense introduced by Ivrissimtzis *et al* [1]: both vertices and face centres in the source lattice map to vertices in the refined lattice. Ivrissimtzis *et al* [1] demonstrate the existence of *dual* $\sqrt{3}$ schemes: where both vertices and face centres in the source lattice map to face centres in the refined lattice. The paper will discuss various characteristics of such schemes (footprint, mask, comparison to $\sqrt{2}$ schemes) including the following issues.

In a dual $\sqrt{3}$ scheme, all refined vertices appear on the edges of the original mesh, as they do in Reif and Peters' simplest scheme [3]. Each refined vertex lies one-third of the way along a source edge. This breaks symmetry: centres of 6-rotation in the source mesh becomes centres of 3-rotation in the refined mesh.

A consequence is that dual $\sqrt{3}$ schemes have two types of triangle in the original mesh. One type of source triangle (A) refines to a triangle rotated by $\frac{+\pi}{6}$; the other (C) to a triangle rotated by $\frac{-\pi}{6}$. Around any regular source vertex, A and C triangles will alternate.

A problem arises with odd-valence extraordinary vertices. There is no way of alternating A and C triangles around such a vertex. There will have to be two triangles of the same type sharing a common edge. Wherever two triangles of the same type have a common edge there is a special case in the connectivity of the refined mesh, which requires careful handling.

Unfortunately an odd-valence extraordinary vertex has an influence beyond its immediate neighbourhood. The inability to alternate A and C triangles around the vertex propagates out into the next ring of triangles, so that there will also be two adjacent triangles of the same type in that ring. This propagation will continue out into further rings of triangles until a second odd-valence extraordinary vertex intercepts the trail of pairs of triangles of the same type.

A result of Nürnberger and Zeilfelder [4] shows that it is possible to label all triangles in any triangular mesh as either A or C so that nowhere are more than two triangles of the same type adjacent. This provides some limitation on the problem, but finding an appropriate labelling is a global optimisation problem rather than a local problem. This non-locality is a serious drawback for a subdivision scheme, and means that no dual $\sqrt{3}$ subdivision scheme is likely to be practical for general geometric modelling.

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Power Expansion of Tangent Lengths in High Accuracy Cubic Hermite Ellipse Approximation

T. Dokken

Keywords: *Approximation, ellipse, implicit, parametric, accuracy.*

In [1] it was proved that proper control of tangent lengths in parametric cubic Hermite interpolation gives errors that are $O(h^6)$. This result is the basis for addressing cubic Hermite interpolation of the ellipse $\mathbf{c}(\theta) = (b\cos\theta, \frac{1}{b}\sin\theta), \tau \leq$

$\theta \leq \tau + \alpha$. It is shown that the error is $O(\alpha^6)$ provided that the tangents $L_1(\alpha)$ and $L_2(\alpha)$ of the Hermite interpolant satisfy

$$L_1(\alpha) = \frac{\alpha}{3} + (\frac{1}{24} + c_1)\frac{\alpha^3}{6} + c_2\frac{\alpha^4}{24} + O(\alpha^5)$$

$$L_2(\alpha) = \frac{\alpha}{3} + (\frac{1}{24} - c_1)\frac{\alpha^3}{6} - c_2\frac{\alpha^4}{24} + O(\alpha^5).$$

It will be shown how different choices of the constants c_1, c_2 and the $O(\alpha^5)$ terms give ellipse approximations with different properties. The approach uses the parameterization $\mathbf{c}(\theta)$ of the ellipse segment to be approximated to parameterize the Hermite interpolant $\mathbf{p}(s), 0 \leq s \leq 1$ with respect to the ellipse segment approximated. Then $\mathbf{p}(s)$, expressed in the Bernstein basis, is inserted into the implicit representation of the ellipse, following the approach in [2]. It should be noted that the lengths of the half axes b and $\frac{1}{b}$ cancel out. The coefficients of the resulting degree 6 Bernstein basis represented polynomial are then power expanded with respect to the angle α. The power expansion of the coefficients is then analyzed to find conditions for establishing the $O(\alpha^6)$ approximation. Detailed information on the approach can be found in [3].

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Stalibity in Periodic Multi-Wavelet Decomposition and Non-Linear Compression/Recovery

Dinh Dung

Keywords: *Stalibity in multi-wavelet decomposition, non-Linear Compression / Recovery.*

Multi-wavelet decompositions and compressions/recoveries based on quantization have been widely applied for image and data processing. Such applications have purpose to efficiently compression/recovery a signal function f from the functionnal coefficients in a multi-wavelet decomposition. There are special multi-wavelet decompositions in which the functional coefficients are values of f at certain points. We discuss non-linear methods for implementing compression/recovery of f from their values, based in such multi-wavelet decompositions. Signal functions to be compressed/recoveried are represented as multivariate periodic continuous functions having a given anisotropic Sobolev or Besov smoothness. We also study the problem of stability in periodic multi-wavelet decompositions which plays an important role in discrete description of Besov quasi-norms in terms of the functional coefficients.

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Face Value Subdivision Schemes on Triangulations

Nira Dyn*, David Levin, Jo Simoens

Keywords: *Subdivision schemes, repeated averaging, convolutions, dual grid.*

Subdivision schemes for values attached to faces of a triangulation, such as texture, are constructed and analysed. The main operations in the refinement step are repeated averaging. The basic averaging operation consists of averaging the face values in the triangulation generating vertices values, which are then averaged with the help of the hexagonal dual mesh. The smoothness analysis is performed in terms of convolutions of the basic limit function, corresponding to one averaging step, with itself.

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Cardinal Interpolation

Karol Dziedziul

Keywords: *Cardinal interpolation, asymptotic formula, best constant, non-homogeneous scaling.*

Let I_h be a cardinal interpolation of polynomial order r, such that for $h > 0$

$$I_h f(x) = \sum_{\alpha \in \mathbb{Z}^d} f(h\alpha)\Phi(x/h - \alpha).$$

Assume that the fundamental function Φ has exponential decay. Let $r > d/p$, $1 \leq p < \infty$. Then there is C such that for all $f \in C_0^r$

$$\|I_h f - f\|_{L^p} \leq Ch^r \sum_{|\beta|=r} \|D^\beta f\|_{L^p}.$$

Asymptotic formula for the error in cardinal interpolation gives the lower estimate of the best constant (such formula exists also for $r < d/p$). But unfortunately the best constant C can be much greater. We introduce the concept of the near optimal best constant for the $f \in C_0^{r+1}$. We control the error in cardinal interpolation more precisely. We would like also to present the non-homogeneous scaling as a method of accelerating the convergence in cardinal interpolation case.

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Minkowski Geometric Algebra of Complex Sets

Rida T. Farouki

Keywords: *Complex sets, Minkowski combinations, Cartesian ovals, interval arithmetic, geometrical optics.*

Algebraic operations, performed on sets of complex numbers, yield remarkably rich geometrical structures with diverse applications and connections to problems in science and engineering. For simple operands, such as circular disks, exact descriptions for their algebraic combinations are available in terms of the Cartesian and Cassini ovals, and higher–order generalizations. Algorithms can be formulated to approximate operations on sets with general (piecewise–smooth) boundaries to a specified precison. This "Minkowski algebra of complex sets" is the natural generalization of (real) interval arithmetic to complex numbers. It provides an elegant two–dimensional "shape operator" language, with connections to mathematical morphology, geometrical optics, and stability analysis of dynamic systems.

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Approximate Moving Least-Squares Approximation: A Fast and Accurate Multivariate Approximation Method

Greg Fasshauer

Keywords: *Radial basis functions, moving least squares approximation, quasi-interpolation, approximate approximation.*

Over the last 25 years or so radial functions have emerged as an increasingly popular method for multivariate scattered data approximation and interpolation. Some of the most commonly used radial functions are multiquadrics, thin plate splines, and Gaussians. For all three of these methods the solution of the scattered data approximation problem often leads to a large dense system of linear equations (on the order of several thousand equations). Frequently this system is also ill-conditioned.

These difficulties have led us to look at moving least-squares approximation based on radial weights. In doing this, one now needs to solve many small linear systems (on the order of three or four equations).

The main result presented in this talk will be an application of the theory of approximate approximations (due to Maz'ya and Schmidt) to moving least squares approximation. This results in a new quasi-interpolation method which does not require any linear systems to be solved. Since one can achieve this with the same numerical accuracy as the usual moving least squares method, a fast and accurate approximation method for multivariate scattered data is available.

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Surface Least Square Approximation: a Shape Preserving Approach

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Keywords: *Scattered data, shape preserving approximation, local support.*

A new approach for fitting surfaces to scattered data is presented. Given the N points (x_i, y_i, z_i) we assume them as measurements of an underlying function $f(x,y)$, where z_i is affected by errors, ; we are looking for an approximating function $F(x,y)$ so that $F(x_i, y_i) \simeq z_i$. In particular the desired values of an approximating surface evaluated at points on a rectangular grid are given. The aim of the work is to take into account more information than just positional values, namely the intrinsic geometric properties of the surfaces. A local fitting scheme is used to minimize a newly-defined objective function which measures some property differences between the unknown surface and its approximation. The objective function is expressed as $\varepsilon = \varepsilon(d_1, d_2)$, where d_1 measures the distance between surface data points and the corresponding approximating surface, and d_2 measures the difference between a mean of Gaussian and of mean curvatures of the surfaces at the data points and the corresponding mean of curvatures on the approximating surface. The first step of the work is to evaluate the above surface curvatures at each data point as well as at grid points. This is performed by means of a local moving least square approximation method to evaluate partial derivatives. Hence a local polynomial least squares surface approximation is defined for each data point, in order to minimize the objective function; for this aim only the data points whose curvatures are close (in a specified sense) to those of the point in question are taken into account, in order to obtain a shape preserving approximation of the surface. Finally a weigthed mean of the values assumed at each grid point G_j by the polynomials corresponding at suitable data points surrounding G_j is assumed as the value of the approximating surface at G_j.

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Mean Value Coordinates

Michael S. Floater

Keywords: *Barycentric coordinates, harmonic functions, mean value theorem, parameterization, morphing, triangulation.*

We introduce a generalization of barycentric coordinates which allows a point in the plane to be expressed as a convex combination of neighbouring points. The coordinates are derived from the Mean Value Theorem for harmonic functions. There are several possible applications in geometric modelling, among them the simplification and improvement of methods for parameterization [1] and morphing [2,3].

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Incremental Selective Refinement on Hierarchical Tetrahedral Meshes

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Keywords: *Tetrahedral meshes, multi-resolution geometric modeling, hierarchical data structures, volume data analysis.*

We consider the problem of modeling large volume data sets, i.e., sets of points spanning a domain D in the three-dimensional Euclidean space and describing a scalar field. This problem arises in several applications, including scientific visualization, medical imaging, simulation, and finite element analysis. A volume data set is often modeled by decomposing its domain D through a tetrahedral mesh with vertices at the data points. We consider a regular a tetrahedral mesh, i.e., a mesh generated by a recursive decomposition on the points of a regular grid. In order to analyze volume data sets of large size, a multiresolution approach can be used. Multiresolution models have been widely used for describing surfaces and two-dimensional height fields (see (De Floriani and Magillo, 2002) for a survey). A multi-resolution model encodes the steps performed by a mesh refinement, or simplification, process in a compact data structure, which encodes the updates iteratively performed on the initial mesh, and from which a virtually continuous set of simplified meshes can be extracted. *Selective refinement*, i.e., the extraction of variable-resolution meshes, is performed by traversing the model and applying different updates among the pre-computed ones. in this way, the resolution (i.e., the density of the cells) of such meshes may be varied in different parts of the field domain (e.g., inside a box, or along a cutting plane), or in the proximity of interesting field values.

In the paper, we consider a regular multi-resolution model based on nested tetrahedral meshes generated by recursive bisection of a tetrahedron along its longest edge. In the computer graphics and finite element literature, there has been a burst of research on nested tetrahedral meshes generated through tetrahedron bisection, or by applying the so-called red/green tetrahedron refinement technique. Nested tetrahedral meshes are used as the domain decomposition technique for multiresolution modeling of regularly spaced volume data sets, or as the basis for scattered data approximation and interpolation. Here, we present an efficient representation for a multi-resolution model based on tetrahedron bisection, that we call a *Hierarchy of Tetrahedra (HT)* consisting of a table of field values and of an implicit description of the forest of binary trees describing the nested decomposition of the cubic domain. We show how to extract adaptive meshes having a resolution varying in different parts of the domain from a HT in an incremental way. To this aim, two issues must be addressed: (i) a conforming mesh must be produced, to avoid discontinuity in the mesh approximation associated with it; (ii) the algorithm must be incremental, i.e., it should be able to locally refine, or coarsen a current mesh by applying a minimal modification while maintaining mesh consistency. In order to find refining and coarsening modifications that maintain consistency, we have developed a worst-case constant-time technique for finding those tetrahedra which forms clusters that must be split, or merged, simultaneously. The worst-case constant-time behavior is achieved through a technique for labeling tetrahedra and through simple and bit-wise operations. We show experimental results in connection with common level-of-detail queries for volume data analysis and rendering, which are specific instances of selective refinement.

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Bimonotonicity Preserving Surfaces Defined by Tensor Products of C^1 Merrien Subdivision Schemes

Françoise Foucher*, Paul Sablonnière

Keywords: *Surfaces, tensor products, Hermite interpolation, Merrien's subdivision scheme, bimonotone interpolation.*

Recently, Merrien and Sablonnière (*Constructive Approximation*, 2002, to appear) have characterized a family of univariate C^1 Hermite interpolants defined by subdivision schemes (Merrien, 1992) having nice shape preserving properties. We extend these properties to bivariate C^1 Hermite interpolants defined as tensor products of univariate ones. We propose some algorithms for the construction of surfaces preserving the bimonotonicity of data (i.e. the monotonicity w.r.t. x and y separately) whatever be the norm of gradients to be interpolated.

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Fast Logical Operations on 2-Simplex Meshes

Maxim Fradkin*, Jean-Michel Rouet

Keywords: *Deformable models, 3-d interaction, mesh intersection and merging, logical operations.*

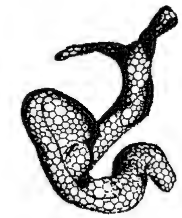
Discrete 3D deformable models are known to be very powerful tools for segmentation of 3D medical images[1][2]. In practical applications, where real-time user interaction is required, it is often necessary to rapidly detect mesh (self-)intersections, efficiently compute the merging of two meshes, or quickly cut parts out of the mesh. In particular, such operations are needed during the generation of initial models, mesh correction, and also when combination of some partial segmentation results is required.

In this paper we discuss a simple and unified approach to handle these operations on 2-simplex meshes or other discrete surface representations (e.g., triangulation). First, two fast low-level operations are defined: *face intersection* (i.e., whether a face is crossed by an edge), and *face relative position* (i.e., whether a non-intersected face is located inside/outside of another mesh). In order to achieve these two operations efficiently, mesh items (faces, vertices or edges) are stored in a bucket data structure. Then, possible mesh (self-)intersections are straight forwardly detected, by checking for existing edge-face intersections. In the best case, using the bucketing technique reduces the intersection algorithm complexity from $O(n^2)$ to $O(n)$. Face relative position is based on ray casting along the outward direction of the face normal, followed by ray-face intersection detection. The face is considered outside of the other mesh if no intersection occurs or if the scalar product between the intersected face outward normal and the ray direction is negative.

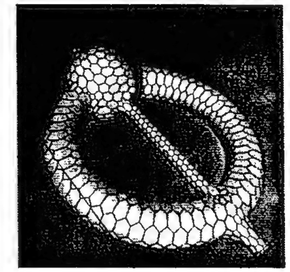
Depending on the required accuracy level (cell or sub-cell), and having the fast intersection detection scheme defined, we propose two approaches for implementing intersection or merging of two meshes, as well as cutting out parts of a mesh. Since each of these tasks is based on the above mentioned fast low-level operations, their resulting complexity is practically also of the same order (i.e. $O(n)$); this allows almost real time mesh interactions. Our approach is illustrated with various examples. For instance, following figures show two intersecting meshes A and B (left), whose intersection curves are used to cut mesh A (middle). The right view shows an example of merged torus, sphere and cylinder.

In conclusion, this paper presents efficient algorithms for logical operations on 2-simplex meshes and their application to semi-automatic segmentation of 3D medical images.

A+B



A−B



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Simplification of Terrains by Minimization of the Local Deformation

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Keywords: *Delaunay triangulation, surface mesh, decimation, local deformation.*

The maps are involved in numerous numerical or graphical simulations. In general, the geometric model (a terrain, for instance) is represented by a triangulation or a grid that may contain several millions triangles or quadrilaterals. The simplification of such maps is thus required, in order to facilitate the storage and the transmission, as well as to make a finite element simulation or the display more efficient.

Several simplification methods for surface triangulations have been developed (see [5] for a survey), that differ mainly by the type of data and the criteria used to measure the deviation from the simplified triangulation to the initial one. Here, we consider the following problem : given $\mathcal{T}$ the *canonical* triangulation of a map (its vertices are the images of the nodes of a regular grid $\mathcal{G}$ of size $n \times m$ from $\mathbb{N}^2$ to $\mathbb{R}^3$ along given heights), find a simplified triangulation $\mathcal{T}_s$ based on a "minimal" subset $\mathcal{V}$ of the vertices of $\mathcal{T}$ in which the "distance" to the map $\mathcal{T}$ is less than or equal to a given threshold value. This gap can be quantified in two ways : continuous or discrete. The continuous measure consists in formulating this gap using the Hausdorff distance from $\mathcal{T}_s$ to $\mathcal{T}$ [1]. The computation

of the Hausdorff measure seems here too expensive. However, the discrete measure consists in considering the gap between the vertices of the map $\mathcal{T}$ and their orthogonal projections on the triangulation $\mathcal{T}_s$ (this being a simplification of the continuous Hausdorff measure and seems more appropriate for terrains simplification). Formally speaking, let $z(p)$ be the height of a node p of the grid and let $u_{\mathcal{T}}$ the vector of dimension $n \times m$ of components $z(p)$, p covering the nodes of the grid and let $v_{\mathcal{T},\mathcal{T}_s}$ be the vector of dimension $n \times m$ of components $z_s(p)$, the height of the orthogonal projection of $P = (p, z(p))$ of $\mathcal{T}$ on $\mathcal{T}_s$. The problem is then to find a subset of minimal cardinality $\mathcal{W}$ of the nodes of the grid $\mathcal{G}$ and a triangulation $\mathcal{T}_s$ based on the vertices images of the nodes in $\mathcal{W}$, such that $\delta_{\mathcal{T}_s} = \|u_{\mathcal{T}} - v_{\mathcal{T},\mathcal{T}_s}\|$ is less than or equal to the threshold δ, where $\| \, . \, \|$ denotes a norm of $\mathbb{R}^3$. As shown by [6], the triangulation $\mathcal{T}_s$ can be considered as the image of the Delaunay triangulation of the nodes of $\mathcal{W}$. Thus, the simplification problem is reduced to finding the nodes of $\mathcal{W}$.

Our approach, unlike methods based on incremental insertion (*greedy insertion*, cf. [4], for instance), aims at constructing the simplified triangulation $\mathcal{T}_s$ (and thus the nodes of $\mathcal{W}$) by removing iteratively the vertices of the initial triangulation $\mathcal{T}$. At each stage, the vertex to be removed is that for which the "current" deformation measure given by $\varepsilon(P) = \max_i d(P_i, \Pi(P))$ is minimal, $d(.,.)$ denoting the distance from a point to a plane, $\Pi(P)$ being the tangent plane at P and P_i, the vertices adjacent to P. The point removal operation consists in removing the triangles incident to this vertex (*i.e.*, the *ball* of the vertex) and to retriangulate the resulting *cavity* using the Delaunay criterion. The simplification procedure stops as soon as the deformation measure becomes greater than a given threshold, at each vertex of the current triangulation. Several examples of large datasets will be provided and detailed (in terms of cpu times and memory requirement) to emphasize the efficiency of this approach.

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Approximate Parameterization of Planar Cubic Curve Segments

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Keywords: *Parameterization, approximation, algebraic curves, rational curves, cubic curves.*

In order to convert algebraic spline curves into a format that can be used in a CAD system, one needs to develop techniques for parameterization.

The existing algorithms for rational parameterization rely mainly on symbolic methods. From classical algebraic geometry, it is known that (in general) exact parameterization by rational functions is not possible. Consequently, the symbolic algorithms for parameterization are applicable just to a very small set of curves. Besides, they will fail if the coefficients of the algebraic curves are given as floating point numbers. So the need of an algorithm for approximate parameterization of algebraic curves is obvious.

As the simplest possible case, we are currently investigating methods for approximately parameterizing planar cubic curve segments.

We transform the problem into Bézier coefficient space with respect to some proper Bézier triangle (see [1]). This allows an identification of cubic curves with points in coefficient space. We deduce an algebraic condition for the rationality of an irreducible cubic curve on its bivariate Bézier coefficients. This results in a hypersurface in Bézier coefficient space, describing the rational cubic curves. We determine a cubic curve $\delta(x, y) = 0$, such that δ is different from f, but approximates the curve segment of f within the triangle. This curve is found by linear Chebyshev approximation. The curves f and δ define the line $f + s\,\delta$ in Bézier coefficient space. The intersection of this line with the hypersurface of rational cubics gives several candidates for rational cubics, which approximate the given curve segment. Useful candidates are detected by a criterion on the Bézier coefficients. After picking an optimal solution, we compute a rational parameterization of the curve segment within the Bézier triangle.

We are planning to generalize the proposed method to curves of higher degree and to the surface case.

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Convex Approximation and Norm Approximation

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Keywords: *Computational geometry, convex polyhedra, Hausdorff metric, duality, high dimensions, approximation algorithms.*

In many computational geometry problems, computing an exact solution is extremely expensive, especially in spaces of high dimensions or embedded with non-euclidean norms. In some cases, an approximation algorithm is sufficient. We will introduce a general method for constructing such algorithms, by replacing a norm on $\mathbb{R}^d$ with a new one which stays close, and which is easier to compute.

This new norm will be defined by the unit ball of its dual norm. To be convenient, this ball has to be a convex symmetric polyhedra, and has to approximate the dual of the initial unit ball. Hence we will deal with the approximation, with respect to the Hausdorff metric, of smooth convex compact sets by polyhedra, taking a special care of spheres and ellipsoids.

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Classification and Regression with Sparse Grids

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Keywords: *Data mining, classification, regression, approximation, sparse grids, combination technique.*

We present a new approach to the classification and regression problems arising in data mining. It is based on the regularization network approach but, in contrast to other methods which employ ansatz functions associated to data points, we use basis functions coming from a grid in the usually high-dimensional feature space for the minimization process. To cope with the curse of dimensionality, we employ sparse grids. Thus, only $O(h_n^{-1}n^{d-1})$ instead of $O(h_n^{-d})$ grid points and unknowns are involved. Here d denotes the dimension of the feature space and $h_n = 2^{-n}$ gives the mesh size. We use the sparse grid combination technique where the classification or regression problem is discretized and solved on a certain sequence of conventional grids with uniform mesh sizes in each coordinate direction. The sparse grid solution is then obtained from the solutions on these different grids by linear combination. In contrast to other sparse grid techniques, the combination method is simpler to use and can be parallelized in a natural and straightforward way. We further extend the method to so-called anisotropic sparse grids, where now different a-priori chosen mesh sizes can be used for the discretization of each attribute. This can improve the run time of the method and the approximation results in the case of data sets with different importance of the attributes.

The method computes a nonlinear classifier but scales only linearly with the number of data points and is well suited for data mining applications where the amount of data is very large, but where the dimension of the feature space is moderately high.

We test the method using standard test problems from the UCI repository and problems with huge synthetical data sets in up to 14 dimensions It turns out that our new method achieves correctness rates which are competitive to that of the best existing methods.

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Curve and Surface Meshing for Finite Element Applications

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Keywords: *Delaunay triangulation, curve discretization, surface meshing, finite element applications, surface simplification, surface optimization, adaptive meshing, surface reconstruction.*

Curves and surfaces are of utmost importance in various engineering applications. In this paper, we are primarily interested in such entities as they play a crucial role in finite element simulations. Actually, this type of numerical simulations involves the solution of a variety of P.D.E.s (partial differential equations) formulated in a bounded domain which need to be discretized by a number of finite elements. Therefore, the first step of such a simulation is the construction of an adequate mesh of the domain of interest.

Two-dimensional domains are meshed based on a discrete definition of their boundaries. A "continuous" boundary is then replaced by a discrete boundary which is a collection of (straight) segments. This leads to finding methods to discretize a curve so as to capture the geometry of the domain as well as to fulfill other requirements related to the envisaged applications.

Three-dimensional domains are meshed using a discrete definition of their surfaces. A "continuous" surface is then replaced by an approximate discretization made of triangles (or quadrilaterals). Meshing a surface leads to discretize the curves bounding this surface and to find methods to constructing a mesh of a surface when the discretization of its boundary curves is provided.

Surface meshing is, in general, a tedious problem. In peculiar, this is not a problem in two dimensions since the geometry of the surface naturally leads to some extent of anisotropy. On the other hand, surface definitions are twofold. One may consider a composite parametric surface (where the geometry is defined by means of a series of CAD patches). One may also consider that the surface definition is a mesh itself which definition being more and more used in practice.

Parametric surface meshing leads to address the construction of a mesh in the corresponding parametric domain. It is a two-dimensional space but inherits from the true geometry. Therefore, we meet an anisotropic mesh construction problem. Delaunay triangulation algorithms are then revisited so as to produce anisotropic triangulations related to the fundamental forms of the surface.

Discrete surface meshing is slightly different since the geometry of the true surface must be obtained using only the given discrete form. Method to carry out this case makes use of local operators (point enrichment, point decimation, edge swapping, point relocation) which are strongly directed by the geometry of the surface after being numerically evaluated.

Curve and surface meshing technologies are discussed in this paper in various contexts and a number of application examples is given to demonstrate the proposed approaches. Examples range from typical finite element applications (in C.F.D., chemistry, solid mechanics, ...) to more exotical applications such as image encoding, surface morphing, decimation, ..., and many others.

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Shapes of Conic Sections

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Keywords: *Shape, curvature, conic section.*

A shape of a plane curve is all information invariant under the group of direct similarities in the Euclidean plane. We consider some differential invariants of conic sections and discuss their properties. The shapes of ellipses, hyperbolas and parabolas are expressed locally by the curvature function with respect to the similarity group. For the same curves, we also compare the logarithmic curvature and the Euclidean curvature in terms of the spherical arc length parameter.

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From a Triangular Mesh to Surfaces Blended by Means of a Convex Combination

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Keywords: *Surface blending, atlas.*

This work deals with modelling a surface of arbitrary topology using the blending of simple surfaces called *primitives.* We propose to blend the primitives by means of a convex combination [2]. This blending implies to match points of primitives and to associate a weight to each of them for the convex combination. The hardest problem concerns the primitive points matching. Previous solutions propose a straight scheme so as to predefine it. The n-sided Gregory patch [3] is used to fill a polygonal hole within a given rectangular patch complex. The primitives are defined as the extension of the n rectangular patches around the hole. Other approaches propose an extension of B-splines to surfaces of arbitrary topology. Grimm and Hughes [4] define one B-spline primitive for each vertex, edge and face of the polyhedral control mesh. Cotrina and Pla [1] define a B-spline primitive by means of a connected set of rectangular faces centered on each vertex (or on each face) in the polyhedral control mesh (after a Catmull-Clark subdivision).

Contrary to those previous approaches, we propose a scheme where the definition of the primitives determines the points matching. Each primitive is defined as a piece of plane which approaches a region defined on a triangular mesh M. The set of regions is a covering of M so that no more than three regions overlap on each point of M. Each region R_i, is made up with a central part belonging only to R_i, and a peripherical part which is shared with other regions. If two regions R_i and R_j overlap, the parts of the two associated primitives which approach $R_i \cap R_j$ must be matched. To do so, each primitive is parameterized on a 2D-domain Ω_i, and appropriate C^1-diffeomorphic transition functions are defined between peripherical parts of these domains Ω_i. Finally, the weights for the convex combination which blends the planar primitives into a smooth surface are given by the building of a partition of the unity on the domains Ω_i.

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(SBR) Surfaces with Base Points

Olivier Gibaru*, Jean-Charles Fiorot

Keywords: *Base point, blow up, (SBR) surfaces, (BR) curves, Newton polygon.*

Given a rational surface S with a base point at (u, v), we demonstrate that the image of this base point is a set of rational curves, a base point being a parameter value for which the rational parametrization takes the value of $(0/0, 0/0, 0/0)$. This result was established by Clebsch [1] and was first reformulated by Warren in the context of geometric design in [3]. If we consider rational surfaces defined in the domain $[0,1]^2$ for simplicity, we show that these rational curves are placed end to end *via* the formalism of massic vectors introduced by Fiorot and Jeannin in [2]. Furthermore we define the relations between the massic vectors of the curves, which are the images of this base point, and the massic vectors of S. We demonstrate a similar result in the case of triangular (SBR) surfaces which have a base point at the vertices of their domain of definition. On the other hand, given n rational curves ($n \geq 5$) which delimite a hole, a set of (SBR) surfaces, defined in one piece on $[0,1]^2$, are constructed with appropriate base points such that these (SBR) surfaces interpolate these n boundary curves. For instance, a seven-sided (SBR) surface with a base point at $(u, v) = (0, 0)$ has been modelled by a set of seven rational curves. In this case, we state that the image of this base point is made up of three consecutives rational curves.

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Recent Achievements in Delaunay Based Surface Reconstruction

Joachim Giesen

Keywords: *Computational geometry, surface reconstruction.*

Surface reconstruction is a powerful modeling paradigm. To create a model of some solid in $\mathbb{R}^3$ one can just sample its surface and apply a surface reconstruction algorithm to the sample. Hence the task in the surface reconstruction problem is to transform a finite sample into a surface model. There are several obstacles a surface reconstruction algorithm might face. The sample could very large, noisy or to sparse too capture all the features of the solid. Another problem often encountered in real world data ist that there boundaries present in the data, i.e. the right model would be a surface with boundaries.

The different approaches to the surfaces reconstruction problem can be divided broadly into two classes. Algorithms in the first class provide implicit surface models while the algorithms in the second class provide explicit surface models. For rendering purposes an implicit surface is likely to be tranformed into a triangular mesh. Most of the explicit approaches to surface reconstruction compute a triangular mesh directly.

The Delaunay triangulation of the sample points and its dual the Voronoi diagram are well studied objects. They have proven to be a valuable tools for providing a solid mathematical framework to phrase surface reconstruction problems as well as for practical implementations.

The achievements of Delaunay based surface reconstruction include a formal statement of a sampling condition for smooth surfaces, topological and geometric guarantees, a new implicit surface model from a sample, boundary and undersampling detection and robust and efficient implementations.

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Spline Curve Approximation and Design by Optimal Control over the Knots

R. Goldenthal*, M. Bercovier

Keywords: *Reparametrisation, interpolation, approximation of a curve, optimal control.*

In [1] Optimal Control methods over reparametrisation for curve and surface design were introduced. The advantage of Optimal Control over Global Minimization such as in [2] is that it can handle both approximation and interpolation. Moreover a cost function is introduced to implement a design objective (shortest curve, smoothest one, etc...). The present work introduces the Optimal Design over knot vectors of non-uniform B-Splines. An interesting aspect is that the interpolation or the approximation matrix might become singular due to invalid knot vector with respect to the current parameterization. A simple regularisation can overcome this numerical problem. Nevertheless in such singular case the geometric description of the null space is given. The following cases were studied :

1. Design: interpolation of a curve.
   Control: minimal curvature or minimal length.
2. Design: approximation of a curve.
   Control: minimal curvature or minimal length.
3. Design: approximation of a curve.
   Control: minimum approximation error.

Finally the case of optimal control over the knots in a primal probelm defined as a global reparametrization is also considered.

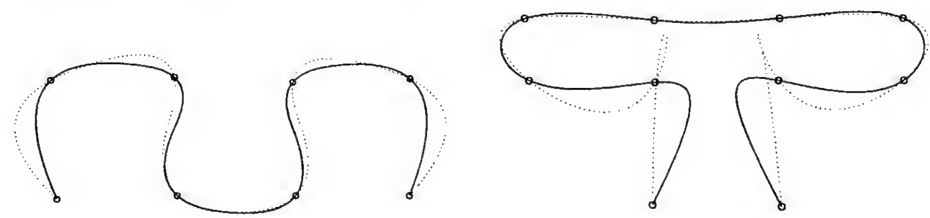

Fig. 1. The dotted curves are the original curves, the dark curves are the result of the Optimal Control process. For both figures the design is interpolation. The left curves are cubic splines, the dark one is controled with approximation of minimal length.The right curves are 5th dergree splines, the dark curve is controled by approximation of minimal curvature.

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Quaternion Splines and Projective Duality.

Aram Gòmez Neri

Keywords: *Quaternions, de Casteljau algorithm, surfaces, animation.*

We present a method to approximate 3D movements using quaternionic functions, staring from diverse set of data, some 2D, taken not necesarily simultaneously. The movement we describe is of a rigid body and we pretend to show it from any poit of view, interpolate it and change the illumination. Data comes from several moving cameras, projective geometry in terms of quaternions is used to determinate the position shape and siza of the object as well as to build virtual images of it "taken" from virtual cameras. Quaternion splines are used to approximate the movement of which only sparse data is known.

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On an Algorithm for Bernstein Polynomials

H. Gonska

Keywords: *Popoviciu algorithm, de Casteljau algorithm, iterated linear interpolation, Bernstein polynomials.*

The talk is related to early work of T. Popoviciu ($\approx$ 1937), P. de Casteljau's famous algorithm ($\approx$ 1959) and simultaneous work of G. Strang ($\approx$ 1962), all showing that Bernstein polynomials can be interpreted as the outcome of n–fold linear interpolation.

It is shown that n–fold linear interpolation may be replaced by p–fold iterated interpolation of appropriate orders, thus revealing the more general structure behind the common evaluation algorithm. The central result is based upon a suitable representation of the classical Bernstein operator.

The present is joint work with Alexandru Lupaş (Sibiu, Romania).

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Dubuc-Deslauriers Subdivision for a Finite Interval

J. de Villiers, K. Goosen*, B. Herbst

Keywords: *Subdivision, interpolatory, interval.*

We consider a method of adapting the Dubuc-Deslauriers subdivision scheme to accommodate sequences of finite length. We will show that this method ensures the convergence of the adapted subdivision scheme and the existence of an associated interpolatory refinable function. Numerical illustrations of the theory are provided.

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Control-Line Curves

A. Ardeshir Goshtaby

Keywords: *Bézier, B-spline, rational Bézier, NURBS.*

Formulations for Bézier, B-spline, rational Bézier, and NURBS curves are revised to use control lines instead of control points. The revised formulations, which will be referred to as control-line curves, will take the form of

$$\mathbf{P}(u) = \sum_{i=0}^{n} \mathbf{L}_i(u) B_{i,k}(u),$$

where $\mathbf{L}_i(u)$ is the ith control line and $B_{i,k}(u)$ is the ith basis function of order k. A control line can be considered as a control point plus a tangent vector. Given a set of control points, the tangent vectors are initially estimated from the control points. The user is then allowed to interactively revise the tangent magnitudes and directions to edit the curve. It will be shown that curves defined by control points are a subset of curves defined by control lines. Therefore, with the revised formulations, some of the properties of the existing curves can be retained, while some of the other properties can be revised to meet an application's particular needs. For instance, if the convex-hull property is a requirement, the magnitude of the tangent vectors may be set to 0 or the tangent directions may be appropriately selected to provide the convex-hull property. However, if convex-hull property is not a requirement and instead it is required for the curve to pass as close as

possible to the control points, the tangent magnitudes and directions may be interactively selected to achieve this.

The formulation for a control-line curve can be decomposed into formulations for two control-point curves, one representing the current curve formulation and the second representing an adjustment to the first. Figure 1 demonstrates differences between control-point and control-line Bézier and B-spline curves. Figures 1a and 1b show the same set of control points but different sets of tangent vectors. The Bézier curve obtained as a result is shown in yellow, while the control-line Bézier curves obtained are shown in blue. Figures 1c and 1d contain the same set of control points but two different sets of tangent vectors. The closed B-spline curve of order 4 obtained as a result is shown in yellow, while the closed control-line B-spline curves of order 4 obtained are shown in blue.

In this paper, formulas for control-line Bézier, B-spline, rational Bézier, and NURBS curves are given and their properties are compared with those of traditional Bézier, B-spline, rational Bézier and NURBS curves.

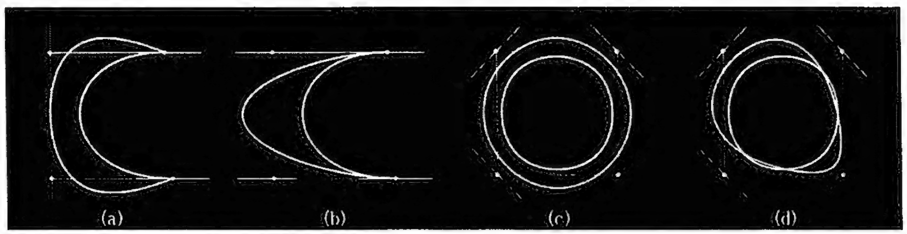

Figure 1. (a), (b) Traditional Bézier (yellow) and control-line Bézier (blue) curves of order 4. (c), (d) Traditional B-spline (yellow) and control-line B-spline (blue) curves of order 4.

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Spectral Methods for Parametrization of 2D and 3D Meshes and Applications in Morphing

Craig Gotsman

Keywords: *Parametrization, morphing.*

We survey methods for parametrizing 2D and 3D meshes based on eigen-analysis of certain matrices derived from the mesh. We demonstrate how these methods may be used in order to smoothly transform ("morph") between two or more 2D objects, such as triangulations, polygons and "stick-figures". The concept of "joint" or "compatible" parametrization will be introduced for this purpose.

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Efficient Rendering of Progressive Polygonal Meshes

C. Gotsman

Keywords: *Polygonam models, vertex buffers, rendering sequences, progressive meshes.*

We review recent developments on accelerated rendering of 3D polygonal models. This includes the use of "vertex buffers" supported in modern graphics hardware. Efficiency is achieved thru the use of locality-preserving rendering sequences which maximize cache coherence. We show how "universal" rendering sequences may be generated and used to efficiently render progressive meshes at all resolutions.

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A Segmentation Process under Interpolation Conditions

D. Apprato, D. Ducassou, C. Gout*, E. Laffon

Keywords: *Deformable models, LSM, interpolation conditions.*

A priori knowledge of interpolation conditions (Lagrange data points, curves etc...) leads to some geometric constraints on the model under study. To avoid the parameterization of the structure, we propose a "level set" approach on deformable models. The solution is obtained by the minimization of a non linear PDE under interpolation conditions.

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Parametric Design using High-Accuracy Hermite Interpolation

Thomas A. Grandine*, Thomas A. Hogan

Keywords: *Hermite interpolation, parametric modeling.*

In 1987, de Boor, Höllig, and Sabin introduced a geometric Hermite interpolation scheme for planar curves. We have recently investigated the applicability of this scheme for design space exploration of air and space vehicles and found it to be a very promising tool, though we found it necessary to modify the scheme to overcome a restriction. This talk will discuss the restriction, the modification, and our experiences with this scheme as a tool for parametric geometric modeling.

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The Length of Subdivision Curves

K. Goosen, J. Gravesen*, J. de Villiers

Keywords: *Subdivision curves, de Rham curve, arc-length.*

A subdivision curve is the limit of a sequence of polygons and it is tempting to use the length of these polygons to estimate the length of the subdivision curve. But it is not certain that the limit curve has a well defined length, and if it has, that the length of the approximating polygons converges towards the length of the limit curve. Finally, it is not known how fast the convergence might be.

The case of a uniform B-spline can be settled using the result of [2] for Bézier curves. This paper also shows that the convergence can be accelerated by considering the length of other polygons. The case of the de Rham curve is studied in [1], but the convergence rate is only established for the quadratic B-spline case.

We show that if a subdivision scheme is uniformly convergent with a continuous limit then the limit curve is rectifiable, i.e., it has a well defined length. In particular, the de Rham curve is rectifiable for a wider range of the parameter than considered in [1].

We have some numerical evidence which suggests that, for some parameter values, there is no definite convergence rate for the length of the polygons.

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Regular 3D Subdivision Methods for Simulation and Visualization

G. Greiner

Keywords: *3D subdivision methods, data compression, numerical simulation, volume data, tetrahedral subdivision.*

Subdivision methods became very popular in 2D for surface modeling. Besides the modeling aspect, subdivision is of interest also for data compression and for numerical simulation. The latter two issues are even more important for 3D-space than just for 2D. Therefore, subdivision for volumes is of some importance.

We will derive several subdivision rules for tetrahedral subdivision of 3D-space, discuss the smoothness and convergence properties of the different schemes. We also present some applications for compression of volume data.

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Approximation with Spline Generated Framelets

R. Gribonval*, M. Nielsen

Keywords: *Wavelet frame, framelets, spline multiresolution, bi-orthogonal spline wavelet, nonlinear approximation, redundant system, adaptive sparse expansions, direct and inverse theorems, Besov spaces.*

We characterize the approximation spaces associated with the best n-term approximation in $L_p(\mathbb{R})$ by elements from a tight wavelet frame associated with a spline scaling function. The approximation spaces are shown to be interpolation spaces between L_p and classical Besov spaces, and the result coincide with the result for nonlinear approximation with an orthonormal wavelet with the same smoothness as the spline scaling function. We also show that, under certain conditions, the Besov smoothness can be measured in terms of the sparsity of

expansions in the wavelet frame, just like the nonredundant wavelet case. However the characterization now holds even for wavelet frame systems that do not have the usually required number of vanishing moments, *e.g.* for systems built through the Unitary Extension Principle, which can have no more than one vanishing moment. Using these results, we describe a fast algorithm that takes as input any function and provides a near sparsest expansion of it in the framelet system as well as approximants that reach the optimal rate of nonlinear approximation. Together with the existence of a fast algorithm, the absence of need for vanishing moments may have an important qualitative impact for applications to signal compression, as high vanishing moments usually introduce Gibbs-type phenomenon (or "ringing" artifacts) in the approximants.

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Preprocessing, Simulation and Visualization on Adaptive Meshes

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Keywords: *Adaptative mesh refinement, 2D and 3D unstructured meshes, nested partitions.*

A simple and efficient algorithm for adaptive mesh refinement of 2D and 3D unstructured meshes with complex boundaries is presented. Starting with an admissible triangulation of the domain the a sequence of nested partitions by local refinement and coarsening is obtained. These partitions are consistent (conform) and stable. The algorithm is based on a set of regular refinement rules, one for each element type, i.e triangles and quadrangles in 2D, and tetrahedra, octahedra, hexahedra, prisms and pyramids in 3D. To compute the closure a full set of irregular refinement rules is used, i.e. there always exists a refinement rule which match any edge and face refinement pattern. For volume meshes with complex boundaries, such as in the case of finite element simulations of the brain shifting problem, a hierarchical surface with subdivision topology is created from the original data. During the adaptive mesh refinement process two criteria can be used, the geometric error of the boundary or the approximation error of the solution in the volume. The position of the new vertices is taken during the refinement from the hierarchical surface. Finally, some computer graphics applications based on these hierarchical grids are discussed.

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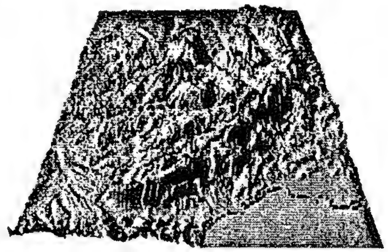
Flexible Approximation of Rough Surfaces with a Fractal Model

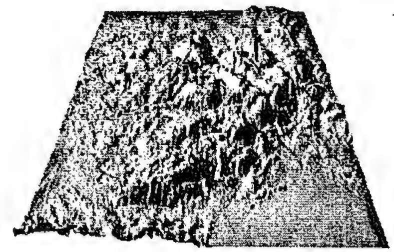
E. Guérin*, E. Tosan, A. Baskurt

Keywords: *Fractal, approximation, surface, rough, projected IFS.*

A wide variety of representation methods have been proposed for modeling surfaces. Unfortunately, these models do not recover rough surfaces, i.e. surfaces defined by continuous functions that are nowhere differentiable. Models that are able to produce rough surfaces are mostly based on random processes. This is the reason why these models are not suitable for approximation. In order to propose an efficient solution to the problem of rough surface approximation, we propose a parametric model based on a deterministic fractal approach. This model, called projected IFS attractors, combines two classical models: a fractal model (IFS attractors) and a CAGD model (free form shapes). A set of control points allows an easy and flexible control of the fractal shape generated by the IFS model and it provides a high quality fitting, even for surfaces with sharp transitions. In [1] and

[2], we proposed an approximation method for curves based on this model. The approximation problem is formulated as a non-linear fitting problem and resolved using a modified Levenberg-Marquardt algorithm. In [3], the extension of this method to surfaces is presented. In this study, we develop methods for the improvement of fitting performances and we show more results on natural surfaces.





The figure above shows a complex natural shape ("Massif Central" French mountain) on the left, and its approximation on the right. Both global aspect and fine details are reproduced in this example. Combining good fitting properties and high flexibility, our approximation method seems to be an interesting approach for rough surfaces approximation and reconstruction.

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A Wavelet Method for fMRI Data Reconstruction

C. Guerrini*, L.B. Montefusco

Keywords: *Funtional Magnetic Resonance Imaging (fMRI), brain imaging, wavelet analysis/synthesis.*

Funtional Magnetic Resonance Imaging is one of the most powerful noninvasive method for observing the activity of living systems. To monitor a dynamic process, as, for example, brain activation, over a short period of time, it requires a series of sequentially acquired MRI images, which present both high temporal and high spatial resolution. Reduced scan imaging is a way of realizing such acquisition that uses some a priori information into the imaging process to reduce the number of dynamic encodings, while achieving high temporal resolution. More precisely, it is composed of one or two complete sets of samples (reference sets), while the other sets (dynamic sets) are acquired at limited spatial resolution. The purpose of this work is to present a new algorithmic procedure for improving the quality of the reduced scan images. It is based on the use of the discrete and continuous wavelet transform and exploits the information enclosed in the reference images to suitably complete the dynamic sets. Numerical results and comparisons with other standard methods are given.

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Efficient Arc Length Computation of Trimming NURBS Curve on a NURBS Surface

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Keywords: *NURBS trimming curve, NURBS surface, arc length.*

This paper presents an efficient algorithm for the computation of a trimming curve's arc length in three dimensional geometric space when the trimming curve and the trimmed surface are both defined using NURBS interpolation.

Let $S(u,v)$ be a bivariate vector-valued piecewise rational function of the form

$$S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} R_i(u)\, R_j(v)\, P_{i,j} \quad 0 \le u, v \le 1,$$

with $R_i(u) = N_{i,p}(u)\,\omega_i / \sum_{k=0}^{n} N_{k,p}(u)\,\omega_k$ and $N_{i,p}$ be the ith non-rational B-Spline basis function of degree p defined over the knot vector

$U = \{\underbrace{0, \ldots, 0}_{p+1}, u_{p+1}, \ldots, u_{r-p-1}, \underbrace{1, \ldots, 1}_{p+1}\}$ and similarly for $R_j(v)$ over the knot vector

$$V = \{\underbrace{0, \ldots, 0}_{q+1}, v_{q+1}, \ldots, v_{s-q-1}, \underbrace{1, \ldots, 1}_{q+1}\}.$$

And let $C_i(t) = (u(t), v(t)) = \sum_{l=0}^{n} R_l(t)\, Q_l \quad 0 \le t \le 1$ be the ith curve in a set of trimming curves defined in the parametric space of S defined over the knot vector

$$T = \{\underbrace{0,\ldots,0}_{l+1}, t_{l+1},\ldots,t_{r-l-1}, \underbrace{1,\ldots,1}_{l+1}\}.$$

The purpose of this paper is to present an efficient arc length computation of a trimming curve based on a subdivision scheme that accounts both for knots in the surface knot vectors and in the curve knot vector. This algorithm does not effectively insert new knots in the curve knot vector but rather subdivides the curve parametric space in spans that are used as bounds in the computation of the arc length.

This subdivision scheme ensures that each span in the trimming curve is defined as a single rational function that can be efficiently manipulated numerically. Spans are determined using the variation diminishing property to localize intersections of the curve with surface iso-lines corresponding to internal knots of the surface. When an intersection with the control polygon is identified, the intersection with the curve is computed by resolution of an implicit formulation of the intersection problem.

Application of the subdivision scheme is demonstrated for the computation of the trimming curve's image arc length in three dimensional geometric space for the discretization of trimmed NURBS surfaces.

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A Hierarchical Structure for Locating Intersections in Large Sets of B-spline Curves

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Keywords: *B-spline curve, intersection, hierarchical structure, bounding box.*

In this paper, a new method for the detection of intersections in large sets of B-spline curves is presented. This model is applied in cartographic processing for the generalisation of isobathymetric lines approximated with B-spline curves. The generalisation problem consists in editing curves in order to ensure security and legibility constraints according to the final map scale. This is due to a large set of input curves (about a thousand curves) defined with up to two thousand points.

In this context, two kinds of intersections are distinguished: visual intersection when two curves are too close at a given scale, and real (self-) intersection. We define an intersection or a self-intersection as an area in which a minimal distance between two segments cannot be guaranteed. We do not apply accurate numerical methods but robust and fast computing geometric methods. Intersections are computed not as single points but as parametric intervals. This makes us possible to process also with singular cases (tangential intersection and overlapping).

Intersections are approximated using a space-partitioning method. First, uniform space segmentation is obtained using a quadtree. Our approach consists in identifying the curve segments related to each cell using a clipping method without the application of a subdivision algorithm [2]. The construction of the quadtree and bounding boxes are done simultaneously: each cell of the quadtree is a bounding box of the curve segments it contains.The segmentation of the cells stops when no intersection exists in a leaf or when an end criterion is reached. In this case, the intersecting curves are marked for correction.

The second phase is to remove the conflicts. For that purpose, a curve can be removed or locally displaced. Therefore, we need to know which control points have to be modified and if a curve exists in the neighbourhood. When modifying a curve, the hierarchical structure can also change because some points can be moved in a neighbouring cell.

The interest of our approach is that fewer operations are made on the curves. Refinement of small parts of a curve is required only in few cases such as small cusps [1] which are difficult to locate. Furthermore, only control point indexes are stored in a cell so that the presented method is fast. It is also reliable for the clipping method can process with singular cases. Our work is illustrated with several examples on real cases.

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Optimal Sizing and Shape Optimization in Structural Mechanics

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Keywords: *Optimal sizing, shape optimization, finite elements, adjoint method.*

We consider an industrial application consisting of the mass minimization of a frame in an injection moulding machine. This frame has to compensate the forces acting on the mould inside the machine and has to fulfill certain critical constraints. The deformation of that frame with constant thickness is described by the plain stress state equations for linear elasticity. If the thickness varies then we use a generalized plain stress state with constant thickness in the coarse grid elements. These direct problems are solved by an adaptive multigrid solver.

The mass minimization problem leads to a constrained minimization problem for a non-linear functional which will be solved by some standard optimization algorithm which requires the gradients with respect to design parameters.

For the shape optimization problem, we assume that the machine components consist of simple geometrical primitives determined by a few design parameters. Therefore, we calculate the gradient in the shape optimization by means of numerical differentiation which requires the solution of approximately 4 direct problems per design parameter. The adaptive solver guarantees the detection of critical regions automatically, and ensures a good approximation to the exact solution of the direct problem.

A different strategy has been implemented for optimal sizing problems. Here, we have to handle hundreds of varying thickness parameters in the optimization problem which makes numerical differentiation non-competitive. The first approach to calculate the gradient quite fast consists in using Automatic Differentiation (AD) of our direct problem solver. This approach works fine for direct solvers for the direct problem but requires huge memory and disk capabilities to handle iterative, and especially adaptive solvers. The other approach consists in writing the total derivative of the functional and get many partial derivatives by solving only one adjoint problem by means of our adaptive solver. Here, a special handling of the geometrical design parameters accelerates the optimization significantly.

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Symmetry Properties in a Subdivision Scheme

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Keywords: *Subdivision schemes, symmetry, smoothness, wavelets.*

To generate subdivision surfaces using subdivision schemes, there are two main classes of meshes: a triangular mesh and a quadrilateral mesh. Recently, there is an increasing interest in $\sqrt{3}$-subdivision, $4-8$ (quincunx) subdivision, and Hermite subdivision schemes in computer aided geometric design. In this talk, we shall unify all these subdivision schemes acting on either a triangular mesh or a quadrilateral mesh under one roof: subdivision schemes with a dilation matrix and a symmetry group. We shall classify all the possible subdivision schemes in a triangular mesh and all the possible subdivision schemes in a quadrilateral mesh. Consequently, new subdivision schemes can be easily obtained by a unified simple algorithm and examples will be presented. Next we shall demonstrate that symmetry of a subdivision scheme can be of great help in analyzing various properties (such as the smoothness exponent and the interpolation property) of the basis function in a subdivision scheme. Finally, we shall discuss how to construct interpolatory subdivision schemes and Hermite interpolatory subdivision schemes via a simple projection method. Related work can be downloaded from the web page at http://www.ualberta.ca/~bhan.

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Ternary and Three-point Univariate Subdivision Schemes

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Keywords: *Ternary, univariate, subdivision.*

Most work in the area of subdivision schemes has considered binary schemes with an even number of control points. We decided to investigate schemes with an odd number of control points, specifically 3-point schemes. This led to a more general investigation of ternary subdivision schemes.

For symmetry reasons, it is obvious that an interpolating binary subdivision scheme which utilizes the closest k points, for k odd, reduces to a scheme which utilizes just the closest $k-1$ points, $k-1$ even. There is thus no 3-point interpolating binary subdivision scheme. Ternary subdivision, on the other hand, does allow for an interpolating 3-point subdivision scheme. A family of such schemes has been shown to exist and have C^1 continuity. Further investigation has led to discovery of a family of interpolating 4-point ternary subdivision schemes which have C^2 continuity.

Investigation of approximating 3-point schemes has led to two interesting subdivision schemes. An approximating 3-point ternary scheme has been found and shown to have C^2 continuity. An approximating 3-point binary scheme, which uses corner-cutting similar in spirit to Chaikin's 2-point scheme $1/4[1,3,3,1]$, is derived and shown to have C^3 continuity.

We have investigated these schemes using the generating function formalism, which lends itself well to deriving sufficient conditions for subdivision schemes to be C^k.

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Sparse Grid Least Squares Fitting Using the Combination Technique

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Keywords: *Sparse grids, least squares, iterative methods.*

The combination method [Griebel et al. 92] is known to be exact for the interpolation problem on sparse grid points and leads to an extrapolation technique for finite elements methods for the solution of PDEs. We will present a new iterative method based on the combination technique for the sparse grid least squares problem. This algorithm generalises the backfitting methods used for additive models [Hastie/Tibshirani 86], and can be interpreted as a block Gauss-Seidel method for singular least squares problems or alternatively as an application of the Kacmarz algorithm. Both theoretical and practical results on the convergence of this algorithm will be discussed.

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Structure from Motion Using a Nonlinear Kalman Filter

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Keywords: *Shape from motion, 3D reconstruction, unscented Kalman filter.*

Suppose one is given a video sequence of an object taken from different angles. Since the sequence contains three dimensional information, it is natural to ask whether it is possible to extract the three dimensional structure of the object from the sequence. Assuming a perpsective transformation, the reconstruction problem is nonlinear and one possible approach is through Kalman filters, or its nonlinear extensions. We propose a multi-estimation algorithm based on one of the nonlinear extensions, the so-called Unscented Kalman filter. It is shown that the algorithm is stable and accurate, even with no prior initialisation.

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Finite Element Approximation with Splines

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Keywords: *Finite elements, B-splines, elasticity.*

We describe the numerical solution of elliptic boundary value problems with weighted extended B-splines (web-splines). These new finite elements possess the familiar approximation and stability properties, but require no mesh generation. For a bounded domain $D \subseteq \mathbb{R}^m$ they are defined as linear combinations of the

scaled translated tensor product B-splines $b_k = b(\cdot/h - k) \quad (k \in \mathbb{Z}^m, h > 0)$, which have some support in D. More precisely

$$B_i = w \sum_{k \in K(i)} e_{i,k}\, b_k,$$

where w is a weight function, incorporating homogeneous essential boundary conditions, and $e_{i,k}$ are suitably chosen coefficients (cf. www.web-spline.de for details).

Ritz-Galerkin approximations with the subspace $\text{span}_i\, B_i$ yield highly accurate numerical solutions with relatively few parameters. This is illustrated for the Lamé-Navier system

$$\begin{aligned} -\text{div}\, \sigma(u) &= f \quad \text{in } D \subseteq \mathbb{R}^3 \\ u &= 0 \quad \text{on } \Gamma \subseteq \partial D \\ \sigma(u)\xi &= g \quad \text{on } \partial D \setminus \Gamma \end{aligned}$$

which models small deformations of elastic solids.

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Application of Knot Modification in Cubic B-spline Design

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Keywords: *B-spline curves, knot modification, envelope, constrained shape modification.*

B-spline and NURBS curves play central role in geometric modeling as de facto standards for the description of free-form objects in CAD. Modifying existing curves is also of great importance, thus several techniques have been developed to reshape a curve based on the direct modification of its data or on the prescription of geometric constraints.

The data structure of a B-spline curve is fairly simple, this consists only of the control points and the knot vector, while in terms of NURBS curves the weight vector has to be specified in addition. Hence shape control methods can modify such curves only by altering these data. The well-known basic possibilities, such as control point repositioning or alteration of a weight can be found in any textbook of the field. Simultaneous modification of multiple elements or different types of data enables more sophisticated shape control, which has been presented in several papers. However, the geometric characterization of knot modifications is not fully explored as yet.

In our presentation we are going to provide some results on the geometric properties of knot modification, along with their application in cubic curve design. We study B-spline curves of degree $k-1$, determined by the control points $\mathbf{d}_j$, $(j = 0, 1, \ldots, n)$ and knots $\{u_j\}$, $(j = 0, 1, \ldots, n+k)$. Modifying a knot value u_i of multiplicity one between its two neighbouring knots, points of $\mathbf{s}(u)$ effected by this modification describe rational curves, that we call paths. For points of arcs $\mathbf{s}_{i-m}, \mathbf{s}_{i+m-1}, (m = 1, 2, \ldots, k-1)$ the degree of these paths in u_i is $k-m$. In order to get an insight into the behaviour of paths we extend their domain on either side, i.e., we let u_i to take values that are less than u_{i-1} or greater than u_{i+1}. An interesting property of these extended paths is that points of paths of the arcs $\mathbf{s}_{i-1}(u)$ and $\mathbf{s}_i(u)$ converge to control points $\mathbf{d}_i$ and $\mathbf{d}_{i-k}$ as u_i tends to $-\infty$ and ∞, respectively.

With the modification of the knot u_i we obtain a one-parameter family of B-spline curves of order $k-1$. This family has an envelope which is also a B-spline curve the degree of which is $k-2$ and it is determined by the same control points $\mathbf{d}_j$ and the knot vector $\{\ldots, u_{i-1}, u_{i+1}, \ldots\}$.

These results on paths and the envelope can also be generalized to knots of higher multiplicity and rational B-spline curves.

We present some examples how to use these paths and envelope for shape modification of cubic B-spline curves subject to positional or tangential constraints.

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Beyond the Classical Theory of Approximation Orders

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Keywords: *Shift-invariant spaces, Sobolev spaces, approximation order, Strang-Fix conditions, polynomial reproduction, refinable distributions.*

We extend the existing theory of approximation orders provided by shift-invariant subspaces of $L_2(\mathbb{R}^d)$ to the setting of Sobolev spaces, provide treatment of L_2-cases that have not been covered before, and apply our results to determine approximation order of multiple linearly independent solutions to refinement equations.

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Geometry Images

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Keywords: *Remeshing, surface parametrization.*

Surface geometry is often modeled with irregular triangle meshes. The process of remeshing refers to approximating such geometry using a mesh with (semi)-regular connectivity, which has advantages for many graphics applications. However, current techniques for remeshing arbitrary surfaces create only *semi-regular* meshes. The original mesh is typically decomposed into a set of disk-like charts, onto which the geometry is parametrized and sampled. In this paper, we propose to remesh an arbitrary surface onto a *completely regular* structure we call a *geometry image*. It captures geometry as a simple 2D array of quantized points. Surface signals like normals and colors are stored in similar 2D arrays using the same implicit surface parametrization — texture coordinates are absent. To create a geometry image, we cut an arbitrary mesh along a network of edge paths, and parametrize the resulting single chart onto a square. Geometry images can be encoded using traditional image compression algorithms, such as wavelet-based coders.

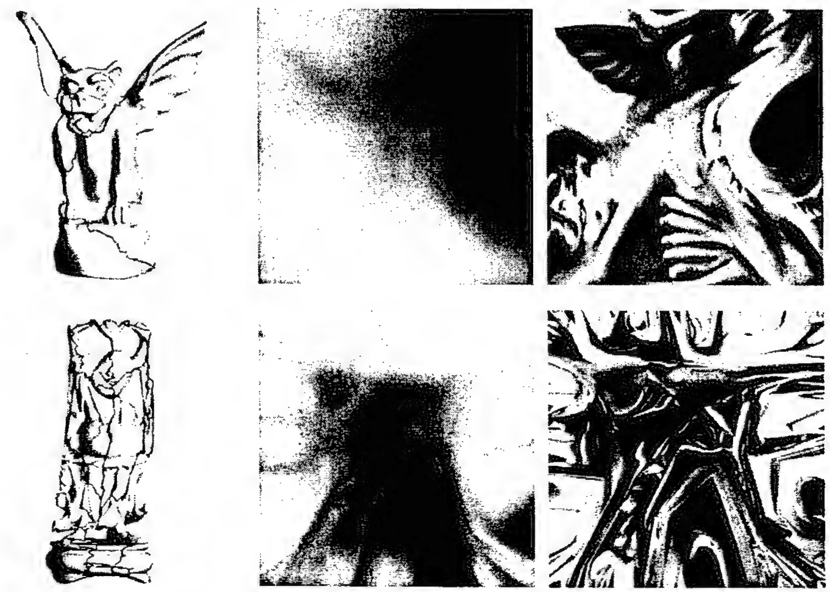

Fig. 1. (a) Cut mesh,
(b) resampled geometry (RGB=(x,y,z)),
(c) normal map (RGB=(n_x,n_y,n_z)).

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Triangulating Unorganized Points

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Keywords: *Triangulation, point clouds, parameterization.*

The problem of reconstructing surfaces from a set of scattered data points arises in many practical situations and we shall focus on the task of triangulating such unorganized point clouds. The problem we consider can be stated as follows: given a set $V = \{v_i\}_i$ of data points $v_i \in \mathbb{R}^3$, find a triangulation $\mathcal{T}$ with these data points as vertices, $V(\mathcal{T}) = V$. Several methods for solving this problem have been developed in recent years, based on different ideas and a detailed survey paper can be found in [3]. A recently presented technique [1] is based on parameterizing the data points and using a Delaunay triangulation of the parameter points in the parameter domain. This method is limited to reconstructing surfaces that are homeomorphic to a disc and we show how it can be extended to handle spherical topology as well. The resulting triangulation can be improved significantly with the approach presented in [2] by optimizing the shape of the triangulation without changing the position or number of vertices but only by modifying the way in which they are connected by triangles.

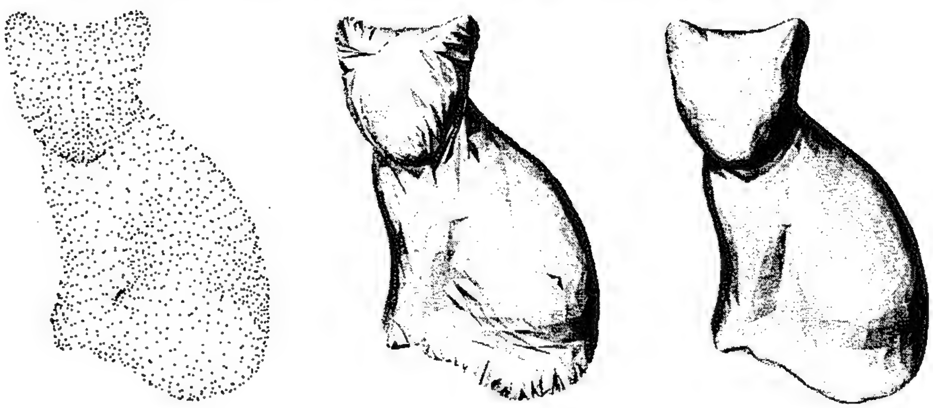

Fig. 1. Point cloud and reconstructed triangulation before and after optimization.

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Quaternion Wavelets and Medical Imaging

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Keywords: *Quaternions, wavelets, medical imaging.*

Our goal is to deal with non rigid surfaces, for example viscous fluids covering objects, interfaces between two fluids (remember; the internal body structures generaly are interfaces between that, e.g.: heart or any internal body structure whit blod and in fact any fluid and any structrure). We then first describe how to approximate a movement via quaternion splines. We also show, in examples like heart movement, quaternionic wavelets as approximating functions using the fact that we have many examples of heart data expressed in terms of wavelets. Those are for example acoustic or electric impulses generated by valves or other parts of the heart. We assign to the image of those parts the movement and the rithm associated to the signal and we interpolate the movements to the other passive parts of the organ what gives a very realistic animation.

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Near-best Spline Quasi-Interpolants on Uniform and Nonuniform Partitions in One and Two Dimensions

P. Sablonnière, M. J. Ibáñez*, D. Barrera

Keywords: *B-spline, discrete quasi-interpolant, near-best quasi-interpolant.*

Let T_s be the uniform partition of $\mathbb{R}^s$, $s = 1, 2$, given by $T_1 = \mathbb{Z}$ and T_2 the three directional (or four directional) mesh of the plane. Lef ϕ be a B-spline on T_s. Different methods to construct discrete quasi-interpolants (dQI) from the values of ϕ at the vertices in $\mathbb{Z}^s$ are known (see [1] and [2], and references therein).

In [3], a method to solve the problem of dQI with minimal infinite norm was proposed, ϕ being a H-spline on the equilateral three directional mesh of the plane, i.e. a B-spline with hexagonal support.

We present some results obtained when this approach is considered with other B-splines relative to T_s. Also, the case of nonuniform partitions is dealt with.

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Adaptive Thinning for Bivariate Scattered Data

Nira Dyn, Michael S. Floater, Armin Iske*

Keywords: *Thinning algorithms, scattered data modelling, piecewise linear interpolation, triangulations, multiresolution methods.*

Thinning algorithms are recursive point removal schemes for scattered data. This talk is concerned with adaptive thinning algorithms for approximating bivariate scattered data by piecewise linear functions over triangulated subsets, as recently suggested in [1]. The resulting point removal schemes can be viewed as methods for *mesh decimation* or *mesh simplification*, see the surveys [3,4]. In contrast to non-adaptive thinning [2], an adaptive thinning strategy depends on both the locations of the data points in the plane, and the values of the sampled function at these points. The proposed adaptive thinning algorithms are greedy: At any removal step, a *least significant* point (which minimizes an estimate of the error incurred by its removal) is deleted. Moreover, the methods generate a hierarchy of subsets of scattered data points, such that the piecewise linear interpolants over the Delaunay triangulations of these subsets are close to the original data. Based on numerical tests and comparisons, two practical adaptive thinning algorithms are recommended, one which is more accurate and another which is faster. Theoretical aspects of adaptive thinning are addressed, and details concerning data structures and efficient coding of these algorithms are discussed.

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Subdivision Rules for n-dimensional Simplicial Complices

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Keywords: *Subdivision, simplicial complices, duality.*

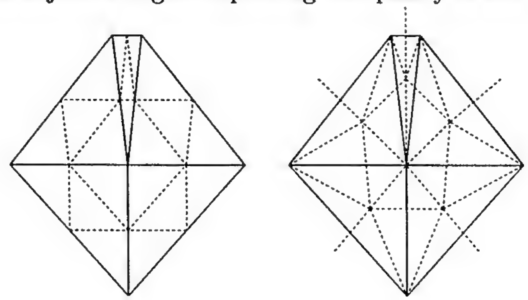
The $\sqrt{3}$-scheme is a method for the progressive refinement of triangular meshes introduced in Kobbelt [3]. Recently, Velho [4] proposed a generalization of the $\sqrt{3}$-scheme to tetrahedral meshes. Here, we introduce an extension of these schemes to n-dimensional simplicial complices.

In Ivrissimtzis and Seidel [2] it was shown that the mesh refinement rule of the $\sqrt{3}$-scheme can be described as the capping of the dual of the initial mesh. That is, we take the dual of the initial mesh, we insert a new vertex at the center of each face and triangulate. Both the duality and the capping operators can be extended in the more general setting of n-dimensional complices, defining this way refinement rules for simplicial complices. That is, in one step of the refinement process, we take the dual of the initial simplicial complex, we insert a new vertex at the center of each cell and triangulate.

This general approach naturally introduces a linear algebra formalism in the study of subdivision. Each vertex is a point of the n-dimensional vector space $\mathbf{R}^n$. Each simplicial complex is the set of convex solutions of a linear equation in $n+1$ variables. The other primitives of the simplicial complex correspond to convex solutions of linear systems of equations and the operators duality and capping are simple linear substitutions.

The analysis of these schemes with linear algebra allows some interesting comparisons with the edgewise subdivision of a simplex introduced in Edelsbrunner and Grayson [1]. In particular, their method keeps the quality of the triangulation constant, see [1], while the schemes proposed here, under certain assumptions, improve the quality of the triangulation. The Figure below shows one refinement step in dimension 2 of the Edelsbrunner and Grayson subdivision and the method proposed here. In practice, they are the Loop subdivision (left)

and the $\sqrt{3}$-scheme (right), without the smoothing step. It is clear that in Loop subdivision the quality of the triangles is constant while in the $\sqrt{3}$-scheme there is an averaging of adjacent angles improving the quality of the triangles.



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Polynomial Curves in Parallel Coordinates : Results and Constructive Algorithm

Zur Izhakiyan*, Alfred Inselberg

Keywords: *Polynomial dual curves, parallel coordinates, approximated surfaces.*

*M*atskewich et al. [1], using the representations of linear flats in parallel coordinates (abbr. ||-coords), presented in this conference a novel way of characterizing and visualizing approximated planes in N-dimensionsal space for any N. Here we extend the representational results of linearly and quadratically defined objects to polynomial curves in order to provide the basis for multidimensional extensions and applications to approximated surfaces. Specifically, we show that the dual image in ||-coords of an algebraic curve of degree n is also algebraic of degree $n(n-1)$ in absence of singular points. In addition, an algorithm based on algebraic geometry, and in particular using **resultants** and **homogeneous polynomials** [2], which constructs the dual image of the curve is provided.

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Approximation with Transformed Radial Basis Functions

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Keywords: *Transformation of variables, radial basis functions of functions, functions of radial basis functions.*

Radial basis functions (RBFs) effectively provide multivariate approximation by use of a sum of one dimensional forms and hence are potentially far more efficient than tensor product forms such as polynomials and splines. In the context of least squares approximation, two particular ways are considered for transforming RBF or approximations by applying a function to either the dependent variable or the independent radial variable .

In the first case, we replace an RBF approximation by a function of an RBF. For this nonlinear problem a general algorithm of Mason and Upton [1] may be used to determine good approximations by a linearisation procedure and we have undertaken numerical tests successfully for a variety of applied functions.

In the second case RBFs may be transformed by a change of radial variable [2], which can be thought of as changing the domain of the original RBF or of defining a new RBF.

For both methods the choice of centres is studied and, for example, standard procedures, such as those based on k-means clustering and discrete Gaussian curvature, need to take account of the functional transformation. The rank of the system matrix is also considered. There is also the potential to extend the methods to a variety of different approximation norms.

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Employing Dilation in RBF Interpolation to Increase Robustity

Michael J. Johnson

Keywords: *Interpolation, approximation order, scattered data.*

One shortcoming of the standard error analysis for RBF interpolation of scattered data is that optimal orders of approximation, with respect to the Sobolev space $W_2^\gamma := W_2^\gamma(R^d)$, have only been obtained for γ in a rather narrow range. For example, in the case of surface spline interpolation (SSI) it is known that optimal orders of approximation are realized on W_2^γ for $\gamma \in [m, m+1/2)$ and are not realized on W_2^γ for $\gamma > m+1/2$. Here m is an integer parameter which specifies the smoothness of the surface spline. For the case $\gamma < m-1/2$, we can show that SSI is unstable on W_2^γ. Thus SSI does not realize optimal orders of approximation on W_2^γ for γ outside the interval $[m-1/2, m+1/2]$.

The situation can be much better if one allows the radial basis function to dilate with h, where h denotes the fill distance from the interpolation points Ξ to the open, bounded domain Ω. To do this, one starts with a fixed radial function ϕ and defines $\phi_h := \phi(\cdot/h)$. Given data $f_{|\Xi}$, the RBF interpolant s is chosen in the form

$$s = q + \sum_{\xi \in \Xi} \lambda_\xi \phi_h(\cdot - \xi),$$

where q is low degree polynomial. Given integers κ, m with $d/2 < \kappa < m$, we use a refined error analysis to identify conditions on the Fourier transform of ϕ (involving the orders of the pole at the origin and the zero at ∞) which ensure that the above described interpolation scheme realizes optimal orders of approximation on W_2^γ for all $\gamma \in [\kappa, m]$. We thus obtain some control over the robustity of the RBF interpolation scheme.

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Meshing for the Computational Science Pipeline: Modeling, Simulation, and Visualization

C. Johnson

Keywords: *Meshing algorithm, simulation, visualization, computational science pipeline, computational field problems.*

Computational field problems; such as computational fluid dynamics (CFD), electromagnetic field simulation, and weather modeling – essentially any problems whose physics can be modeled effectively by ordinary and/or partial differential equations–constitute the majority of computational science and engineering simulations. Such problems often require a researcher to apply diverse skills in confronting problems involving very large data sets, three-dimensional complex geometries, large scale computing, and visualization of the models and results.

Currently, meshing, simulation, and visualization are often seen as separate, distinct, non-integrated processes. However, if we integrate meshing, simulation, and visualization, one can often reduce the overall complexity of, and increase computational efficiency in, the computational science pipeline. With such integration, one must think about algorithms and software development in a new way and consider the entire computational science pipeline.

In this talk, I will present recent research on meshing algorithm and software development for the computational science pipeline using examples from large scale computational field problems.

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Spatial Geometric Interpolation

G.D. Vassilatos, A.I. Ginnis, P.D. Kaklis*

Keywords: *Spatial interpolation, Hermite interpolation, geometric boundary conditions, Frénet-frame continuity, quintic Bézier curves.*

This work deals with the problem $\mathcal{P}(\mathcal{D}; GBC)$ of constructing a third-order Frénet continuous curve, that interpolates a spatial data set $\mathcal{D}$ containing N points, $\mathcal{D} = \{\mathbf{P}_i, i = 1, ..., N\}$, and satisfies *Geometric Boundary Conditions (GBC)*, i.e., given Frénet frame, curvature and torsion, at the boundary points $\mathbf{P}_1$ and $\mathbf{P}_N$. An analogous two-dimensional problem, namely interpolating a planar point-set with GBC being equivalent to specifying the boundary unit-tangent vector and curvature at $\mathbf{P}_1$ and $\mathbf{P}_N$, is thoroughly investigated in Ginnis & Kaklis (2002).

In the present paper we construct a two-parameter family of quintic Bézier solutions to the Hermite version, $N = 2$, of the above problem. The properties of this family are classified, according as a boundary curvature is zero and/or a boundary binormal is orthogonal to the unit-tangent at the other boundary point. The free-parameter pair is selected by minimizing the L_2–norm of the $2^{nd}/3^{rd}$ parametric derivative of the curve. Next, in conjunction with alternative techniques, enabling the estimation of Hermite data (Frénet frame, curvature, torsion) from a given point-set, the solutions of $\mathcal{P}(\{\mathbf{P}_i, \mathbf{P}_{i+1}\}; GBC)$, $i = 1, ..., N-1$, are pasted together for providing a solution to $\mathcal{P}(\mathcal{D}; GBC)$.

The numerical performance of the proposed solutions, for both the Hermite ($N = 2$) and general ($N > 2$) versions of $\mathcal{P}(\mathcal{D}; GBC)$, is tested and discussed against data taken from a helix, a bicylindrical and an exponential curve.

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Greedy Approximation and Multivariate Haar System

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Keywords: *Weak tresholding greedy algorithm, unconditional basis, multivariate Haar system.*

Let $\mathcal{X} = (x_n, n \geq 1)$ be a normalized basis in a Banach space X, and let $\tau = (\tau_n, n \geq 1)$, $0 \leq \tau_n \leq 1$, be a sequence of real numbers (called *a weakness sequence*). For $x \in X$, $x = \sum_{n=1}^{\infty} a_n x_n$, let $(n_k, k \geq 1)$, be a sequence of indices satisfying $|a_{n_1}| \geq \tau_1 \max_n |a_n|$, and $|a_{n_k}| \geq \tau_k \max_{n \neq n_1, \ldots, n_{k-1}} |a_n|$ for $k \geq 2$. By *weak tresholding greedy algorithm* (WTGA) with respect to a basis $\mathcal{X}$ and a weakness sequence τ we mean an approximation algorithm whose m-th step is given by $G_m^{\tau}(x; \mathcal{X}) = \sum_{k=1}^{m} a_{n_k} x_{n_k}$. When $\tau_n = 1$ for all n, then the corresponding WTGA is called just *tresholding greedy algorithm* (TGA). Let us note that WTGA considered here is an adaptation of the weak greedy algorithm considered in [4], [2] or [5].

The talk is based on paper [1]. At first, a necessary and sufficient condition for convergence of WTGA with respect to a fixed normalized unconditional basis $\mathcal{X}$ in a Banach space X and a weakness sequence τ is given. Application of this result gives a simple necessary and sufficient condition for a weakness sequence τ, under which WTGA with respect to τ and the multivariate Haar system $\mathcal{H}_p^d$ in $L^p[0,1]^d$, $1 < p < \infty$, is convergent. Here, by the multivariate Haar system $\mathcal{H}_p^d$ we mean the system consisting of tensor products of the univariate Haar system $\mathcal{H}_p$.

The order of approximation by WTGA will also be discussed.

The second part of the talk concerns TGA with respect to the multivariate Haar system $\mathcal{H}_p^d$. It is well known that in case of $d \geq 2$ and $p \neq 2$, the multivariate Haar system $\mathcal{H}_p^d$ is not a greedy basis in $L^p[0,1]^d$, i.e. for this system, in general, the order of approximation provided by m steps of TGA for individual functions is not comparable to the best m-term approximation with respect to $\mathcal{H}_p^d$ (a suitable example can be found in [3]; see also [6]). However, for some functions from $L^p[0,1]^d$, the approximant given by m steps of TGA may give the order of approximation comparable to the best m-term approximation. In this direction, we discuss the structure of *greedy configurations* from the multivariate Haar system, i.e. subsequences $\mathcal{G}$ of $\mathcal{H}_p^d$ with the following property: if $f \in L^p[0,1]^d$ and all non-zero coefficients of the basic expansion of f with respect to $\mathcal{H}_p^d$ come from $\mathcal{G}$, then the order of approximation of f by m steps of TGA is comparable to the best m term approximation of f with respect to $\mathcal{H}_p^d$.

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Tensor-Border Nets and Patches

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Keywords: *Sabin net, tensor-border patch, subdivision, blending.*

Tensor-border nets are a natural generalization of Sabin nets for higher degree boundary curves and higher order continuity. Tensor-border patches represent the corresponding generalization of Sabin's[3], Hosaka-Kimura's[1], and Loop and DeRose's[2] construction. The construction is best understood by splitting the modelling process in to two parts: net creation and patch selection.

For the net creation, we consider a new subdivision scheme compatible with rational surface patches. For the patch selection, we explain how G^1 and G^2 tensor-border patches (both rational and piecewise polynomial) are efficiently represented and how they generalize known representations. Special attention is paid to fair blending of natural quadrics and tori as part of a freeform spline scheme.

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Arc-Length Parameterized Spline Curves for Real-Time Simulation

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Keywords: *Spline, real-time, simulation, arc-length, parametric.*

Parametric cubic splines are the curves of choice for many applications of computer graphics. They are widely used in computer animation and virtual environments to determine motion paths. In many applications it is essential to efficiently relate parameter values to the arc length of the curve. Current approaches to compute arc length or to construct an arc-length parameterized curve are too inefficient to use in real-time applications. This paper presents a simple and efficient technique to generate approximately arc-length parameterized spline curves that closely match spline curves typically used to model roads in high-fidelity driving simulators.

Our technique avoids the high cost of arc-length parameterization by generating a new curve that accurately approximates the input curve and is approximately parameterized by arc length. A substantial advantage of this approach is that the heavy burden of computation is pushed into off-line preprocessing steps. The payoff is that on-line computations can be computed very quickly. Moreover, the precision of the on-line computations can be predetermined during the preprocessing stage by selecting the granularity of the approximation curve. The major on-line cost incurred by increasing the accuracy of the approximation is the added space needed to store a larger number of knot points and spline coefficients.

The approximation curve is computed in three steps. First, numerical integration is used to compute the lengths of the component cubic pieces of the original spline curve $Q(t)$. The individual lengths are summed to determine the total length of $Q(t)$. The second step is to find $m + 1$ points equally spaced along $Q(t)$ using a bisection method. The third step is to compute a new spline curve using the equally spaced points as knots. The result is an approximately arc-length parameterized piecewise spline curve divided into m cubic segments.

The derived curve is an approximation in two senses: (1) the shape of the derived curve approximately matches the shape of the input curve and (2) the derived curve is approximately arc-length parameterized. Our paper examines errors in the match to input curve, errors in the deviation from arc-length parameterization, and the relationship between these two types of error. We present techniques to solve each of the three steps in building an approximation curve and report the results of computational experiments with input curves representative of those used in highway design. The results show a 10 fold reduction in the maximum match error and a 5 fold reduction in the maximum deviation from arc-length parameterization with each doubling in the number of segments in the derived curve.

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Image Registration Using Parametric Surfaces and Pixel Diffusion

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Keywords: *Global motion estimation, multimodal images,multisensor images, gradient methods, image alignment.*

This paper presents an energy minimization approach to the registration of significantly dissimilar images, acquired by sensors of different modalities. The proposed algorithm introduces a robust matching criterion by aligning the locations of gradient maxima. The alignment is formulated as a diffusion process where a set of pixels is driven over a parametric surface evaluated using a single image. The second image's gradient maxima locations are used only for initialization. Thus, an implicit matching criterion is achieved while utilizing full spatial

information and without resulting to invariant image representations. We are able to robustly estimate affine and projective global motions using coarse to fine processing even when the images are characterized by complex space varying intensity transformations. These cause current state-of-the-art algorithms to fail. Finally, we present results of registering real multi-sensor and multi-modality images using affine and projective motion models.

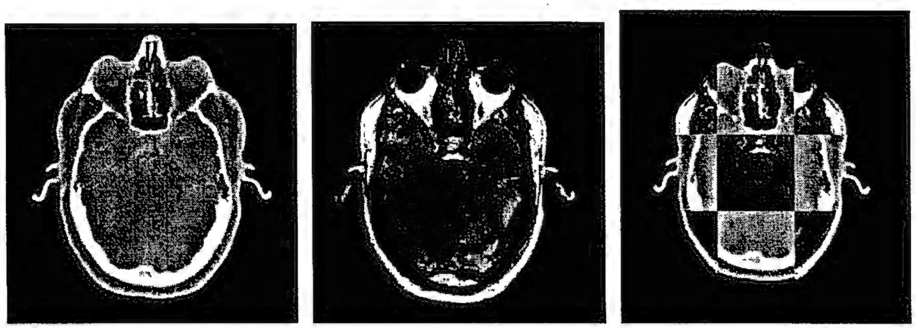

Fig. 1. Registration results for multi-modality CT and MRI images.

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Shape Preserving Approximation with Large Sets of Scattered Data

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Keywords: *Radial basis function, discrete Gaussian curvature, affine invariant metric, quadratic form.*

In many areas of approximation, it is necessary to construct an approximating surface that accurately reproduces the shape of the data. When the underlying surface is given at a large, finite set of discrete, scattered locations in the xy-plane, this can be difficult due to storage and computational problems. In this paper, we extend the research presented by Morandi and Sestini [1], where scattered data are approximated using radial basis functions with compact support and a reduced set of centres. In [1], the approximation to the data $F = \{f_i\}_{i=1}^m \in \Re$ defined at the abscissae points $X = \{\mathbf{x}_i\}_{i=1}^m \in \Re^2$ is of the form $s(\mathbf{x}_i) = \sum_{j=1}^n b_j \Phi_j(\mathbf{x}_i - \mathbf{x}_j^*)$, for $i = 1, \ldots, m$, where, $\Phi_j(\mathbf{x}_i) := \phi_j(r) = (1-r)_+^3(1+3r)$, and $r = \frac{\|\mathbf{x}-\mathbf{x}_j^*\|_2}{\delta_j}$. Here, because the number of centres used in the above linear form aims to be much less than the number of data points, the centre dependent scale factor δ_j is necessary to ensure that the abscissae domain is properly represented by the basis functions. However, this form of scaling produces problems when the data are considered to be functions of independent variables of different scales. Problems arise in this situation because the Euclidean norm is not invariant under different scalings of the coordinate components of the abscissae.

We propose to replace the scaled Euclidean norm above with an affine invariant metric, namely a quadratic form [2]. A method for choosing the appropriate quadratic form is given and it is shown how this is geometrically equivalent to replacing the radial basis functions (RBFs) by hyper-ellipsoid basis functions (EBFs). We show how the accuracy of the approximated surface can be improved by using this generalised form. We further describe extensions to the discrete Gaussian curvature algorithm described in [1] for selecting a optimum set of shape preserving centres. The paper examines results obtained using Gaussian, Wendland and thin plate spline radial basis functions applied to data sets representing a sequence of three stages in the application of fissure sealant to a molar tooth. These data sets, obtained via a Laserscan 3D coordinate measuring machine, have in excess of 10,000 data points.

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Geometric Properties of Bases and Statistical Estimation Problems

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Keywords: *m-terms approximation, coding, nonparametric estimation, wavelets.*

We start with a completely general atomic setting of an unconditional basis in a (quasi-) Banach space and we show that representation of the spaces of m-terms approximation as Lorentz spaces is equivalent to the verification of Jackson and Bernstein inequalities, and that either of these properties is equivalent to one Temlyakov property. The proof is very direct, and especially does not use interpolation theory.

This context is very flexible and allows us to give very simple evaluations of the metric entropy of quite different spaces as well as provide very simple universal codings.

It is also a very convenient setting to perform statistical nonparametric estimation. In this context, we are able for instance, to answer the following question : Given an arbitrary rate of convergence and a thresholding procedure, what is the maximal set of functions where the procedure has this particular rate of convergence ? The answer especially points out the connection between statistical estimation and approximation theory.

In a second time, we consider the case where the norm of the quasi- Banach space is given by a quadratic variation formula and thanks to a condition called p-reverse inequality, we obtain a representation theorem. This quadratic variation framework is in fact very rich and contains for instance the cases of Lebesgue and Hardy spaces. We also consider the cases of 'weighted' Hardy and Lebesgue spaces when the weight belongs to a Muckenhoupt class and the basis is a wavelet basis. We finally consider the case of 'warped wavelets', which appear as composition of usual wavelets with a scaling function, and behave, under appropriate conditions, as well as ordinary wavelets. This provides a new example of bases perfectly well adapted to statistical estimation and approximation.

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Near-Interpolation with Arbitrary Constraints

S. Kersey

Keywords: *Spline curves, interpolation, near-interpolation, smoothing, approximation.*

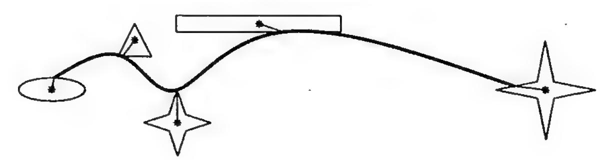
This talk concerns the nonlinear programming problem

$$\inf_f \left\{ \int_a^b |D^m f(t)|^2 \, dt : f(t_i) \in K_i \right\}$$

over spline curves $f : [a,b] \to \mathbb{R}^d$ of order $2m$ that meet sets $K_i \subset \mathbb{R}^d$ at data sites t_i, i=1:n. The problem is convex when K_i are convex, such as closed balls; the problem is quadratic when the constraints are given by quadratic functions. Here, K_i are only assumed to be star-shaped with respect to prescribed points z_i. In this case we define the constraints by inequalities $\mu_i(f(t_i)) \leq 1$ with μ_i the Minkowski functionals

$$\mu_i : x \longmapsto \inf\{\alpha > 0 : x \in z_i + \alpha(K_i - z_i)\}.$$

Approximate solutions are computed using a simple fixed point iteration. We also consider methods to improve the parametrizations (i.e., choices for the data sites t_i), and we allow tangent constraints (when $m \geq 2$). Due to the nonlinearities in the problem, computing minimizers precisely can be an arduous task. Therefore, the goal here is to obtain "good" curve fits at minimal expense.



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Medial Surface Reconstructions on Profiled Interpolated NURBS Surfaces

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Keywords: *Medial surface, surface interpolation, profiled surface, surface discretization, automatic blocking.*

This paper presents a medial surface reconstruction method based on a NURBS surface subdivision algorithm. The subdivision algorithm uses reconstructed attachment curves on the surface to split the surface and isolate the features in the surface associated to the leading edge. The attachment curves are build using a curvature criterion at the leading edge to minimize curve oscillation. These curves allow a simpler construction of the medial surface. Attachment curves also serve as the basis of an automatic blocking process to support volumic discretization of the surrounding space of the profiled surfaces.

The paper will discuss both the attachment curve reconstruction algorithm and the medial surface construction method, and will present an application of the method to the automatic blocking and discretization of hydraulic turbine blades for analysis purposes.

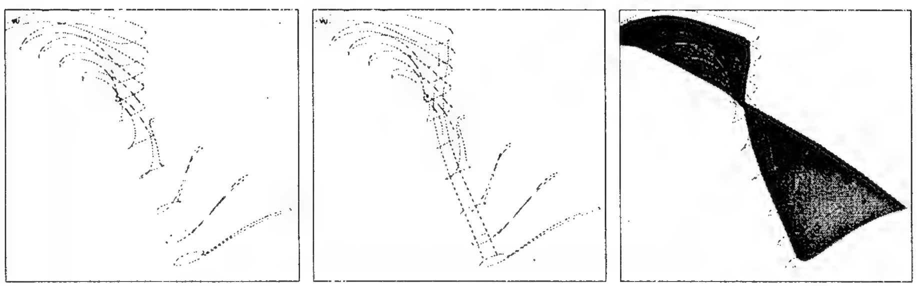

Fig. 1. Attachment curve reconstruction steps.

Figure 1 illustrates the main reconstruction steps of the attachment curves and medial surface: discrete data from three dimensional sections describing the blade surface are first oriented and interpolated (Fig. 1a), non-oscillating attachment curves are constructed (Fig. 1b), and the medial surface is reconstructed (Fig. 1c).

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Marching on Triangulated Domains

R. Kimmel

Keywords: *Minimal geodesic problem, triangulated domains, Voronoï diagrams, robotic navigation, texture mapping.*

The speaker will review a computationally optimal numerical answer to the question of how to compute the shortest path between two points on a surface, also known as the 'minimal geodesic problem'. A numerical technique for solving Eikonal equations on triangulated curved domains is introduced. It provides an optimal scheme for computing geodesic distances and thereby solving the minimal geodesic problem. Next, we show who to use the method to compute

1. minimal geodesics on weighted curved domains,
2. Voronoi diagrams and offset curves on surfaces, and
3. applications of the technique to areas like
   - 3.1. 3D shape reconstruction in computer vision,
   - 3.2. path planning in robotic navigation,
   - 3.3. texture mapping in computer graphics, and
   - 3.4. bending invariant signatures for isometric surface classification.

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Geodesic Curvature Flow on Parametric Surfaces

Alon Spira, Ron Kimmel*

Keywords: *Geodesic curvature flow, surface, manifold, level sets.*

The motion of curves and images in R^2 has been researched extensively. There are many applications in image processing and computer vision, such as, scale space by linear and nonlinear diffusion, image enhancement through anisotropic diffusion and image segmentation by snakes. The level sets formulation [7] has provided good means to implement these flows. Extending these motions to manifolds embedded in spaces of higher dimensions can be beneficial in computer graphics, computer vision and image processing.

In this paper we demonstrate a numerical scheme to extend the curvature flow on the plane [3] to the geodesic curvature flow on parametric surfaces. The flow is implemented by back projecting the flow on the parametric surface to the parameterization plane, calculating the flow on the plane by the level sets method and then mapping it back to the manifold. This technique has been used before for manifolds that are graphs of functions ($\{x, y, z(x, y)\}$), in order to find shortest paths [5,6] and to construct an intrinsic scale space for images on surfaces [4].

A different approach has been previously used for general manifolds [2,1]. It consisted of implicitly representing both the manifold and the curve on it as level sets of functions in R^N (N - the dimension of the embedding space). This approach has several drawbacks, which are alleviated by our method. Unlike this method, our method does not require the extension of the manifold and the curve or data on it to functions in R^N. The calculations are done implicitly on the parameterization plane and not in R^N, which might be computationally prohibitive for $N > 3$. Finally, our method is not restricted to manifolds that can be represented by a level set, and we can thereby handle more general manifolds, such as self intersecting ones.

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Homogeneous Newton-Raphson Methods for Complex Roots

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Masanori Kimura*, Fujio Yamaguchi

Keywords: *Geometoric Newton-Raphson method, homogeneous processing.*

The geometric Newton-Raphson method is commonly used to compute intersections in curves and surfaces. When it is applied to rational curves and surfaces, however, it often raises various types of uncertainties.

These uncertainties with the processing of rational curves and surfaces are classified into three cases. The first is divergence case, where the parameter value diverges and is overflowed. The reason follows below: The increment of parameter t can be represented as $\delta t = -f(t)/\dot{f}(t)$. This means that determination of intersection points of a curve and an geometric element using the method is equivalent to finding the root(s) of polynomial equation $f(t) = 0$ by the same method. Thus we shall call $f(t)$ an equivalent function. When the curve is rational, then $f(t)$ is also. Supposing $f(t)$ is written as $f(t) = (a_n t^n + \cdots + a_3 t^3 + a_2 t^2 + a_1 t + a_0)/(a_{nw} t^n + \cdots + a_{3w} t^3 + a_{2w} t^2 + a_{1w} t + a_{0w})$ then $\lim_{t \to \pm\infty} f(t) = a_n/a_{nw}$. This shows that when we apply the conventional method to a rational curve, we have always a horizontal asymptote in addition to vertical ones in $f(t)$ and thus have always the possibility of the algorithm breakdown when the parameter happens to approach in the horizontal asymptote region. Therefore we deal with a rational curve, not in $\mathbf{p}(t) = [X(t)/w(t)\ Y(t)/w(t)]$, but in $\mathbf{P}(t) = [w(t)\ X(t)\ Y(t)]$, that is, we homogenize the coordinates of the rational curve. This technique can completely get over the convergence uncertainty inherent in the processing of rational curves and surfaces.

The second is non-local uniqueness case, where the initial parameter values corresponding to a solution are not in a unique continuous range, but scatter in different ranges. This problem is also really serious. To improve this characteristic essentially we homogenize the parameter as well as coordinates such that $t = \sigma/(\eta + \sigma)$. This gives a degree of freedom to determine the parameters η and σ. Then we control these parameters such that $(\delta\eta)^2 + (\delta\sigma)^2$ is minimized.

The third is complex root case, where the parameter value oscillates around a complex root and can not converge because the algorithm does not assume a complex case. Therefore we generalize our geometric Newton-Raphson method by applying the two homogenization techniques mentioned above, so that it can handle complex root cases. We put the parameter as $t = t_r + t_i \cdot i$ where t_r and t_i represent real and imaginary parts, respectively. The equivalent function is also complex, that is, $f_r(t_r, t_i) + f_i(t_r, t_i) \cdot i$. In the complex case, the equivalent function does not have an asymptote, and thus there does bot occur divergence problems. As mentioned above, we had better homogenize parameter as well. We put σ as $\sigma = \sigma_r + \sigma_i \cdot i$. The equivalent function this case is $HF_r(\sigma_r, \sigma_i, \eta) + HF_i(\sigma_r, \sigma_i, \eta) \cdot i$. We determine the increments $\delta\sigma_r, \delta\sigma_i, \delta\eta$ such that $(\delta\sigma_r)^2 + (\delta\sigma_i)^2 + (\delta\eta)^2$ is minimized.

The homogenization technique above presents a superb local uniqueness property as well as almost complete convergence property in the complex root cases. The homogeneous Newton-Raphson methods presented above have been borne out by many experiments.

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Approximation by Generalized Sampling Series

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Keywords: *Shannon sampling series, generalized sampling series, window function, kernel function.*

We shall consider approximation problems by the generalized sampling series given by (see [1] and references cited there)

$$(S_W f)(t) := \sum_{k=-\infty}^{\infty} f(\frac{k}{W})s(Wt-k) \quad (t \in \mathbb{R}; W > 0)$$

for a uniformly continuous function $f \in C(\mathbb{R})$. The Whittaker-Kotelnikov-Shannon sampling series is defined by the kernel $s(x) = sinc(x) := \sin \pi x/(\pi x)$. We shall consider a band-limited kernel function s defined via a window function $\lambda \in C_{[0,1]}$, $\lambda(0) = 1$, $\lambda(u) = 0$ $(|u| \geq 1)$ by equality ([2], [3])

$$s(t) := \int_0^1 \lambda(u)\cos(\pi t u)du.$$

We shall find the order of approximation by those sampling series and evaluate exact values of their operator norms in cases when the window functions are associated with names like Rogosinski, Lanczos, Hann, Blackman etc.

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Efficient High Quality Rendering of Point Sampled Geometry

L. Kobbelt

Keywords: *Hierarchical representation, point sampled geometry, rendering algorithm, 3D transformations, shading, surface splatting.*

We propose a highly efficient hierarchical representation for point sampled geometry that automatically balances sampling density and point coordinate quantization. The representation is very compact with a memory consumption of far less than 2 bits per point position which does not depend on the quantization precision. We present an efficient rendering algorithm that exploits the hierarchical structure of the representation to perform fast 3D transformations and shading. The algorithm is extended to surface splatting which yields high quality anti-aliased and water tight surface renderings. Our pure software implementation renders up to 12 million Phong shaded and textured samples per second and about 4 million anti-aliased surface splats on a commodity PC. This is more than a factor 10 times faster than previous algorithms.

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Parametrising Complex Triangular Meshes

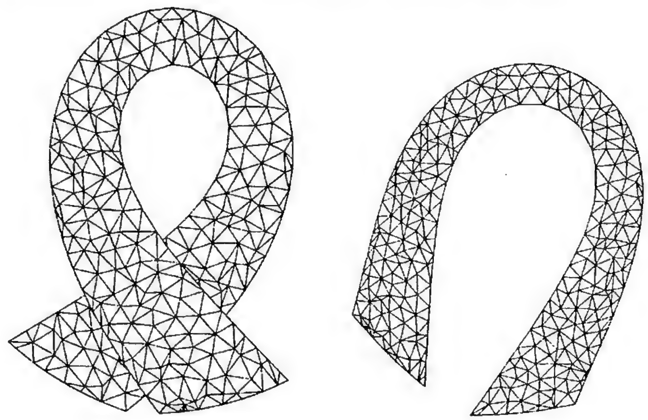
Géza Kós*, Tamás Várady

Keywords: *Surface fitting,parameterisation,triangle meshes,decimation.*

Parametrising polyhedral meshes, in particular triangular meshes, is an important topic of computer-aided geometric design. Many applications in surface fitting, multiresolution modelling, texture mapping or surface remeshing need an efficient algorithm which maps a 3D polyhedral mesh to the plane, preserving the topological structure and minimising distortion.

Our goal is to construct parametrisations for a wide set of triangular meshes in such a way that the map of each set inherits the geometric properties of its source as much as possible. The core of our new algorithm is a variant of Floater's method [1,2]. However, the main effort was put on resolving topological limitations and improving computational efficiency for large meshes. Special procedures are introduced to fill holes and map the 3D boundaries — with convex and concave segments — into the domain plane. The speed is drastically increased using a special, manifold-based decimation algorithm.

Several examples are given to demonstrate the algorithm.



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Solving Linear–Quadratic Elliptic Control Problems by Wavelet Techniques

Angela Kunoth

Keywords: *Control problems, elliptic PDEs, boundary control, wavelets, iterative solution.*

For the numerical solution of a control problem governed by an elliptic boundary value problem with boundary control, wavelet techniques are employed. A quadratic cost functional involving natural norms of the state and the control is to be minimized. Firstly the constraint is formulated as a saddle point problem, allowing to handle varying boundary conditions explicitly.

Deviating from standard approaches, then biorthogonal wavelets are used to derive an equivalent infinite discretized control problem which involves only ℓ_2–norms and –operators. Classical methods from optimization yield the corresponding optimality conditions in terms of two weakly coupled (still infinite) saddle point problems for which a unique solution exists. For deriving finite–dimensional systems which are uniformly invertible, stability of the discretizations has to be ensured. This together with the ℓ_2–setting circumvents the preconditioning problem since all operators have uniformly bounded condition numbers independent of the discretization.

In order to numerically solve the resulting (finite–dimensional) linear system of the weakly coupled saddle point problems, fully iterative methods are presented and their convergence is shown. The one class of methods can be viewed as inexact gradient or conjugate gradient schemes, consisting of an outer iteration which alternatingly picks the two saddle point problems, and an inner iteration to solve each of the saddle point problems, exemplified in terms of the Uzawa algorithm. The other class is an All–In–One Solver applied to the normal equations.

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Difference Method for Constructing Shape-Preserving Spline Approximations

B. Kvasov

Keywords: *Multipoint boundary value problem, discrete hyperbolic tension spline, discrete thin plate tension spline, shape-preserving approximation.*

The spline theory is mainly grounded on two approaches: the algebraic one (where splines are understood as smooth piecewise functions, see, e.g. [5]) and the variational one (where splines are obtained via minimization of quadratic functionals with equality and/or inequality constraints, see, e.g. [4]). Although less common, a third approach [2], where splines are defined as the solutions of differential multipoint boundary value problems (DMBVP for short), has been considered [1]. Even though some of the impotrant classes of splines can be obtained from all three schemes, specific features sometimes make the last one an important tool in practical settings. We want to illustrate this fact by the examples of shape-preserving hyperbolic and thin plate splines.

For the numerical treatment of DMBVP we replace the differential operator by its difference approximation. This permits us to avoid calculating hyperbolic functions and to easily find mesh solution whose extension will, however, be a discrete hyperbolic spline with continuous differences instead of derivatives. Recently discrete generalized splines and GB-splines have been studied in [3]. We consider the basic computational aspects of this approach and illustrate its main advantages.

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Using Cubic Interpolation for the Extraction of Isosurfaces from Tetrahedral Grids

Ulf Labsik*, Günther Greiner

Keywords: *Tetrahedral grids, isosurface extraction, Bézier curves.*

For numerical simulations tetrahedral grids are often used to discretize 3D scalar fields. The extraction of isosurfaces is one of the standard techniques for the visualization of such a scalar field. To determine an intersection point of an edge with the isosurface linear interpolation is used. But as shown in Figure 1 the quality of the extracted isosurfaces is poor. We therefore present a technique for cubic interpolation along the edges of unstructured grids. This approach is based on tetrahedral Bézier patches. By determining 4d normal vectors for every vertex of the grid, we can define Bézier points for each tetrahedral patch. With the help of this cubic Bézier representation we are able to interpolate along the edges and to extract smoother isosurfaces from the given data set, as shown in Figure 1.

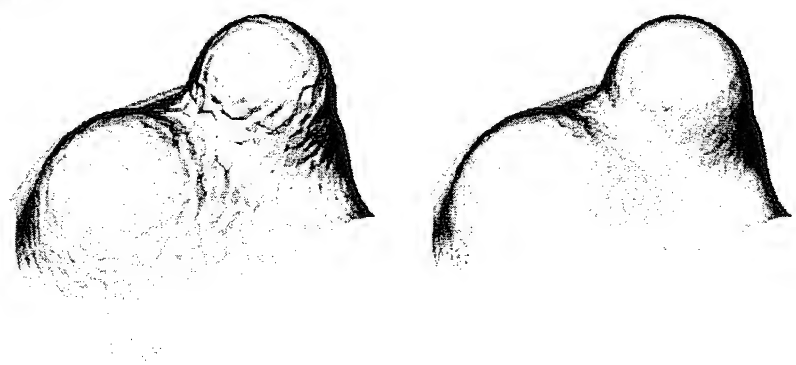

Fig. 1. Left: isosurface extracted from a tetrahedral grid using linear interpolation. Right: isosurface for the same value, but extracted using cubic interpolation.

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B-Spline based Grid Generation and Grid Representation for H-adaptive Finite Volume Discretizations

K.-H. Brakhage, F. Bramkamp, Ph. Lamby*, S. Müller

Keywords: *Grid generation, B-splines, multiresolution schemes, finite volume discretization.*

Concentrating on the geometric aspects, we present the main conceptual ingredients of the new flow solver QUADFLOW for large scale simulations of compressible fluid flow and fluid-structure interaction. In order to keep the size of the discrete problems at every stage as small as possible for any given target accuracy we employ a multiresolution adaptation strategy. This adaptation method gives rise to refined meshes of quadtree respectively octree type. Consequently, a finite volume scheme for fairly general cell partitions is required. In particular, hanging nodes must be handled. For this purpose, the locally adapted grid is treated as a fully unstructured mesh with arbitrary polygonal/polyhedral control volumes in two and three space dimensions, respectively. The second important ingredient is the generation of such meshes along with the information needed by the flow solver at any stage of a dynamical calculation. We employ a multiblock structure, where the mesh in each block results from evaluating a parametric mapping from the computational domain into the physical domain. Such mappings can be based on B-spline representations in combination with well established concepts from CAGD. The quantitites to be updated in time are the relatively few control parameters of the underlying B-spline, while mesh points on any level of resolution can be computed efficiently by function evaluation. Experience has shown that the adaptive code reacts sensibly to small disturbances in the flow field, for example introduced by microscopic wriggling in the B-spline surface description. Therefore we have implemented appropriate fairing techniques. Furthermore, well established methods of high quality grid generation based on partial differential equations, e.g. elliptic or hyperbolic grid generation

methods, can be integrated easily into our concept due to new algorithms for fast interpolation and approximation on tensor product structures.

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Legendre-Bernstein Basis Transformations and their Applications

Byung-Gook Lee*, Yunbeom Park, Jaechil Yoo

Keywords: *Legendre, Bernstein, basis transformations, Bézier curves, degree reduction.*

We study the relationship of transformations between Legendre basis and Bernstein basis. Using the relationship, we present the efficient method to degree elevation and degree reduction of Bézier curves which are widely used in CAGD.

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A Fast Algorithm for Solving a Linearized SVM Problem

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Keywords: *Epsilon insensitive loss function, regularization, simplex tableau.*

The epsilon insensitive loss function was developed by Vapnik [1] for constructing curves and surfaces using support vector machine (SVM) regression. These problems are most commonly formulated as quadratic programming problems and solved by adopting several specialized algorithms, which need in the order of m^2 memory and time resources. In this paper, we adopt Vapnik's metric to construct a general model for the epsilon insensitive approximation problem from which ℓ_1 approximation is a special case. Unlike the standard SVM approach, we replace the usual ℓ_2 regularization term in the objective function with the ℓ_1 norm. This allows the formulation of a linear programming problem (LPP) and a solution process based on the simplex method. Although standard simplex approaches can be used to solve the LPP, it is desirable to have specialized algorithms that can exploit certain features in the structure of the problem to produce fast, accurate results. In this paper, we present an algorithm which exploits the special features and solves the LPP in an extremely efficient way. The savings gained for both computational effort and storage requirement are illustrated through several experimental problems.

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A Family of 4-Points Dyadic High Resolution Subdivision Schemes

Daniel Lemire

Keywords: *Subdivision schemes, Fourier transform, CAGD, wavelets, Lagrange interpolation.*

By using temporary placeholders on a dense grid, we generalize the 4-point dyadic cubic Deslauriers-Dubuc scheme. Interpolated values require 2 steps to stabilize as they are first interpolated on a coarse scale through a tetradic filter and then on a finer scale using a dyadic filter. The interpolants are C^1 and can be chosen to reproduce polynomials of degree 4. These generalized interpolatory subdivision schemes have minimal support and no additional memory requirement.

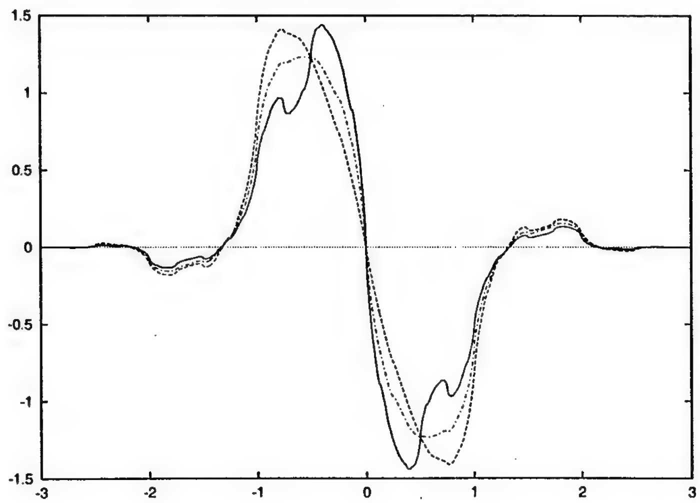

Fig. 1. First derivatives of the fundamental functions of three different 4–points dyadic high resolution subdivision schemes of order 3. One of the schemes (dot-dash curve) is the 4–points Deslauriers-Dubuc scheme.

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Recognition and Reconstruction of Translational Surfaces and Ruled Surfaces

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Keywords: *Reverse engineering, surface fitting, active contour, translational surface, ruled surface, B-spline surface.*

A translational surface is generated by a translatory motion of a base curve along a second base curve, whereas a ruled surface is traced out by a one-parameter motion of a straight line. Both surface classes have many important applications, e.g. in architectural design. Here, a method is presented to detect if a given geometric shape (parametric surface, subdivision surface, point cloud) is close to a translational or ruled surface. An algorithm for the computation of the approximating surface is given. This new approach is based on a more general active contour model to surface approximation. The resulting surface adapts to the model shape iteratively with the help of local quadratic approximants of the squared distance function.

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Fast Evaluation of Radial Basis Functions on the Sphere

Rick Beatson, Jeremy Levesley*, Will Light

Keywords: *Fast evaluation, radial functions, spheres.*

In [1] Freeden et. al. outline a method for fast evaluation of a linear combination of zonal functions $s(x) = \sum_i \alpha_i \phi(x \cdot y_i)$, $x, y_i \in S^2$, the 2–sphere. The sphere is hierarchically divided into panels. The panelling defines a *near field*, N_x, and a *far field*, F_x, for x. Then, we split the summation for s is split into two pieces:

$$s(x) = \sum_{\{i: y_i \in N_x\}} \alpha_i \phi(x \cdot y_i) + \sum_{\{i: y_i \in F_x\}} \alpha_i \phi(x \cdot y_i).$$

The first sum on the right hand side typically has a small number of terms and side is evaluated explicitly. For the second term ϕ is approximated by a polynomial of degree M (typically $\gg 10$),

$$\phi(t) \approx \sum_{k=0}^{M} b_k P_k(t),$$

where P_k is the degree k Legendre polynomial. The second sum can then be computed:

$$\begin{aligned} s(x) &\approx \sum_{k=0}^{M} b_k \sum_{\{i: y_i \in F_x\}} \alpha_i P_k(x \cdot y_i) \\ &= \sum_{k=0}^{M} \gamma_k b_k \sum_{j=-k}^{k} Y_{kj}(x) \sum_{\{i: y_i \in F_x\}} \alpha_i Y_{kj}(y_i), \end{aligned}$$

using the addition formula for the spherical harmonics Y_{kj}. (Here γ_k is a normalisation constant).

The fast method relies on a once and for all computation of the *moments* $\sum_{\{i: y_i \in \text{panel}\}} \alpha_i Y_{kj}(y_i)$ for all of the panels in the subdivision of the sphere.

We describe a refinement of Freeden's algorithm which involves a more sophisticated approximation of the basis function ϕ, depending on the size of panels and the distance between them. By doing this we look to reduce the degree of the polynomial approximation to under 10, thereby gaining in evaluation costs. A careful explanation of the algorithm as applied to the circle will aid understanding in the more complicated 2–sphere setting.

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On the Relations between Piecewise Polynomial and Rational Approximation in $L^p(\mathbb{R}^2)$

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Keywords: *Bivariate piecewise polynomials, bivariate rational functions, approximation spaces, nonlinear approximation.*

In the univatiate case there are certain equivalnces between the nonlinear approximation methods that use piecewise polynomials and those that use rational functions. It is known that for certain parameters the respective approximation spaces are identical and may be described as Besov spaces. The characterization of the approximation spaces of the multivariate nonlinear approximation by piecewise polynomials and by rational functions is not known. In this work we compare between the two methods in the bivariate case. We show some relations between the approximation spaces of piecewise polynomials defined on n triangles and those of bivariate rational functions of total degree n which are described by n parameters. Thus we compare two classes of approximants with the same number cn of parameters. We consider this the proper comaprison between the two methods.

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Construction of Non-Uniform Stationary Subdivision Schemes

Adi Levin

Keywords: *Non-uniform subdivision, approximation.*

A stationary subdivision scheme works by applying the same subdivision (or refinement) operator at each level of resolution. In non-uniform stationary subdivision, the refinement operator uses different rules at different places in the domain (e.g. boundary rules, crease rules).

I will present a method for constructing schemes of this kind, such that they generate polynomials up to a prescribed degree. This property is necessary and sufficient for achieving a desired approximation order, and it is a necessary condition for smoothness.

I will demonstrate the method by constructing a univariate scheme which is interpolatory on one side of the origin and non-interpolatory on the other side. In the bivariate case, I will show how to construct a C^2 scheme that operates on a tri-quad grid, i.e. half of the grid is quadrilateral, and half of it is triangular.

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Smoothness Analysis of Quasi-Uniform Subdivision Schemes

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Keywords: *Non-uniform subdivision, smoothness analysis, bivariate.*

We study the smoothness of quasi-uniform bivariate subdivision. A quasi-uniform bivariate scheme consists of different uniform rules on each side of the y-axis, far enough from the axis, some different rules near the y-axis, and is uniform in the y direction. For schemes that generate polynomials up to degree m, we derive a sufficient condition for C^m continuity of the limit function, which is simple enough to be used in practice. It amounts to showing that the joint spectral radius of a certain pair of matrices has to be less than 2^{-m}. We also relate the Hölder exponent of the m-th order derivatives to that joint spectral radius. The main tool is an extension of existing analysis techniques for uniform subdivision schemes, although a different proof is required for the quasi-uniform case. The same idea is also applicable to the analysis of quasi-uniform subdivision processes in higher dimension. Along with the analysis we present a 'tri-quad' scheme, which is combined of a scheme on a triangular grid on the half plane $x < 0$ and a scheme on a square grid on the other half plane $x > 0$ and special rules near the y-axis. Using the new analysis tools it is shown that the tri-quad scheme is globally C^2.

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Least Squares Conformal Maps

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Keywords: *Parameterization, triangulated surfaces, texture mapping.*

We present LSCM (Least Squares Conformal Maps), a new quasi-conformal parameterization method for triangulated surfaces, based on a least-squares approximation of the Cauchy-Riemann equations. We show the following properties:

- Our criterion minimizes angle deformations and non-uniform scaling. The objective function is a quadratic form, that can be efficiently minimized.

- We prove that this quadratic form is definite positive, therefore the existence and uniqueness of the minimum is ensured.

- The borders of the charts do not need to be fixed. Therefore, large charts with arbitrarily shaped borders can be parameterized.

- We prove that the orientation of the triangles is preserved, which means that no triangle flip can occur. However, as with other methods where the border is not fixed, overlaps may appear, when the boundary of the surface self-intersects in texture space.

- We prove that the result is independent of the resolution of the mesh, therefore, irregularly sampled objects can be correctly handled.

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Medial Axis Homotopy

André Lieutier

Keywords: *Medial axis, homotopy.*

The main result presented here is the homotopy equivalence of any bounded open subset of R^n with its Medial Axis. Because the Medial Axis is used in several fields of research including Computational Geometry and Mathematical Morphology, it may be of interest to be able to rely on general theoretical results on its nature, beyond the smooth case.

Precisely, we call here *Medial Axis* of a bounded open set O, the set of points of O who have at least two closest point on the boundary of O.

Two spaces X and Y are said to have the same homotopy type if there are continuous maps $f : X \mapsto Y$ and $g : Y \mapsto X$ such that $g \circ f$ is homotopic to the identity map of X and $f \circ g$ is homotopic to the identity map of Y.

Having the same homotopy type entails the one to one correspondance of arcwise connected components and the isomorphism of their respective fundamental groups, as well as higher order hmology groups.

The topology of the Medial Axis transform has been studied by G.Matheron in the general case : he proves the connectivity of the closure of the medial axis of a connected open set and is unable to prove the connectivity of the medial axis itself even if it "seems rather plausible" . This connectivity is a corollary of our result.

The homotopy equivalence has been proved by F.E. Wolter in the case of an open set with a piecewise C^2 boundary in the plane and in the case of an open set with a C^2 boundary in R^n. Of course, this result is also known in the case of the complement of a finite number of points in R^n : in this case, the Medial Axis is the Voronoi diagram. These are paticular cases of our result.

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Interpolation by Translates of a Basic Function

Will Light

Keywords: *Radial basic function, translates, error estimates, interpolation.*

Let $\psi : \mathbb{R}^n \to \mathbb{R}$ be a given function. Our talk will concentrate on interpolating data at m points $x_1, \ldots, x_m$ by m translates of this basic function ψ. These translates will always involve the interpolation points themselves, so that the interpolant has the form

$$s(x) = \sum_{i=1}^{m} a_i \psi(x - x_i), \qquad x \in \mathbb{R}^n .$$

The most common form of ψ is a radial basic function so that $\psi(x) = \phi(|x|)$, where $|\cdot|$ is the usual Euclidean norm in $\mathbb{R}^n$. Given data $d_1, \ldots, d_m$ in $\mathbb{R}$, we then solve the equations

$$d_j = s(x_j) = \sum_{i=1}^{m} a_i \psi(x_j - x_i), \qquad j = 1, \ldots, m.$$

Why is this scheme so beloved by people handling multivariate data? Well, the folklore has it that

1 the interpolant exists independent of the location of the data points (except that the data points must be distinct), and so can handle scattered data
2 some special choices of ϕ lead to interpolants with very nice properties
3 very good asymptotic error estimates are available. These error estimates are indicative of the performance of the error in a typical application
4 the method is simple to programme, and can handle data in high dimensions with no difficulty at all.

Of course, these statements are precisely what I claimed them to be – folklore. The practical outflow from some of them is very different. I will discuss in broad terms the significant developments which have taken place over the last 10 years or so, which have culminated recently in translating the folklore into reality. Part of this progress has been underwritten by some very clever mathematics, and part of it by advances in computing technology. I hope to answer the following questions:

1 why do the equations have a unique solution, and where does the basic idea for this method originate?

2 what are some of the useful indications that the error performance of these methods is so good, and how are they couched mathematically?

3 how does one go about inverting the interpolation matrices that result, particularly if one has a very large number of data sites?

In my talk I will aim chiefly to appeal to the non-expert, although I hope there will also be a number of items of interest to the experts.

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Fast Penetration Depth Computation Using Dual-Space Expansion, Hierarchical Refinement and Rasterization Hardware

Ming C. Lin

Keywords: *Computational geometry, graphics hardware, image-space computations, dynamics simulation, geometric modeling, robotics.*

Penetration Depth (PD) is defined as the minimum translational distance to separate two intersecting rigid polyhedral models. We present two classes of PD algorithms, one for convex models based on local estimations and one for non-convex models using a global approach:

(1) DEEP: Dual-space Expansion for Estimating Penetration depth is an incremental algorithm for estimating PD between 3D convex polytopes. It incrementally seeks a "locally optimal solution" by walking on the surface of the Minkowski sums.

(2) Our general PD algorithm for non-convex models uses a combination of object-space and image-space techniques. It computes the pairwise Minknowski sums of decompsed convex pieces, perform closest-point query using rasterization hardware, and refines the estimates by a combination of hierarchical culling, model simplification, and object-space walking.

We highlight its performance on complex models and demonstrate their application to rigid-body dynamic simulation (Image 1), tolerance verification (Image 2) and haptic rendering (Images 3 and 4).

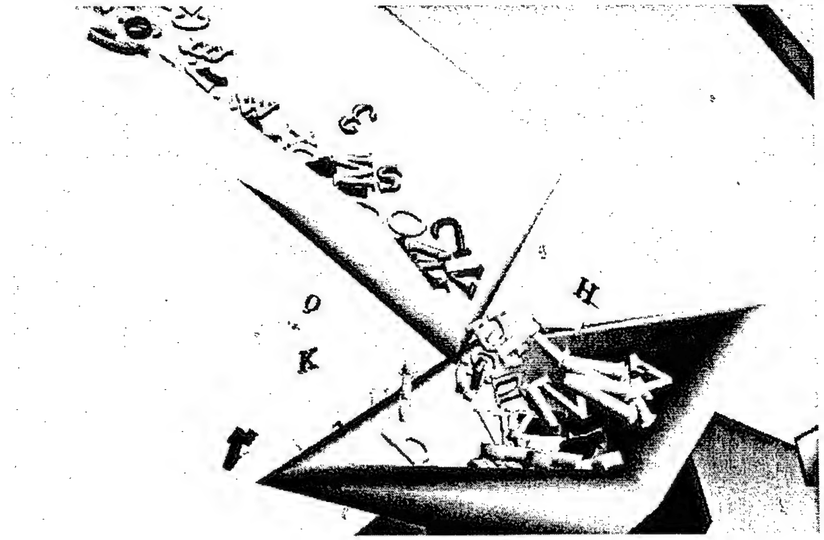

Fig. 1. Image 1: Application to Rigid-Body Dynamic Simulation.

Our algorithm is used to perform smarter time stepping in a dynamic simulation. A rigid-body simulation of 200 models of letters and numerical digits falling onto a structure consisting of multiple ramps and funnels.

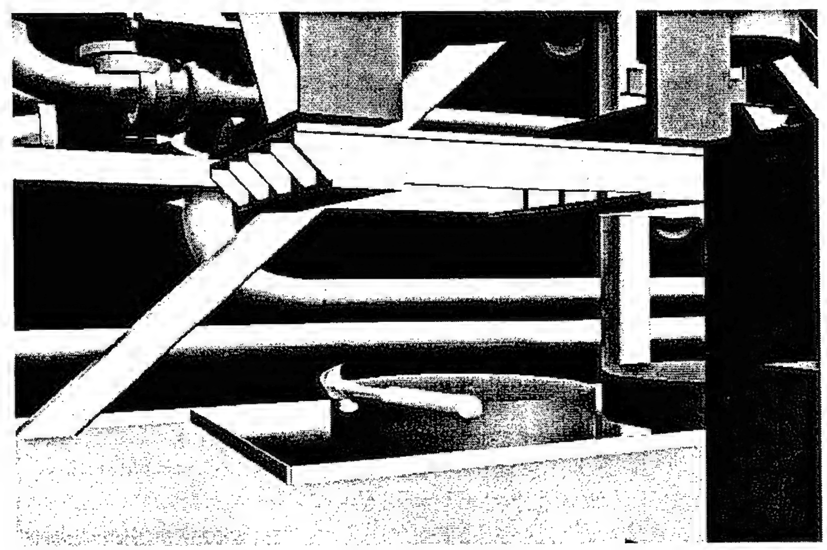

Fig. 2. Image 2: Tolerance Verification Scenario.

Our algorithm is used to check for positive and negative distances between a

tool (yellow hammer, about 400 units long) and the nearby structures along a planned maintenance path in a virtual machine room. The amount of penetration is indicated as a negative number on the lower left corner of the scene.

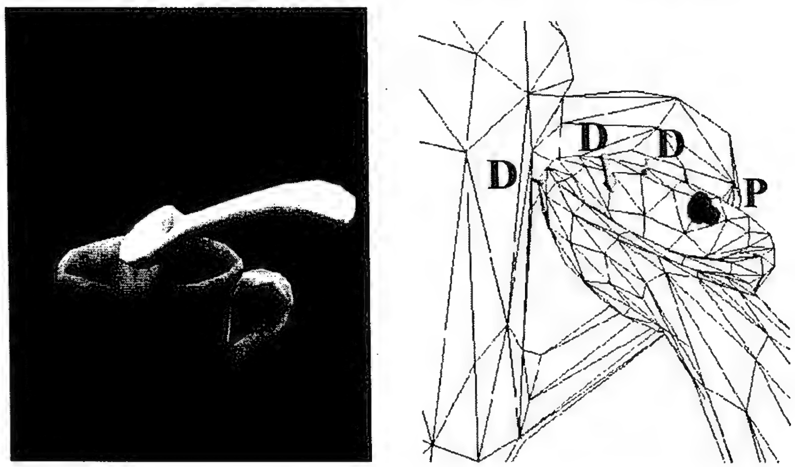

Fig. 3. Images 3 and 4: A Contact Scenario for Cup and Spoon Models.

The forces are computed at clustered contacts for both disjoint (D, green arrows) and penetrating (P, red arrow) situations.

1. `http://www.cs.unc.edu/ geom/DEEP/`
2. `http://www.cs.unc.edu/ geom/6DOFLCC/`
3. `http://www.cs.unc.edu/ geom/PD`

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On Wavelet Coefficients of Functions

Jüri Lippus

Keywords: *Wavelets, modulus of continuity.*

We study the coefficients of wavelet and multiresolution-type expansions of functions with a given majorant of the modulus of continuity.

We say that the function $\omega(\delta)$ is a majorant if $\omega(\delta)$ is non-decreasing, $\omega(0) = 0$ and $\omega(\delta_1 + \delta_2) \le \omega(\delta_1) + \omega(\delta_2)$. Let $Lip(\omega; C)$ denote the set of all continuous functions for the moduli of continuity of which we have the estimate $\omega(f, \delta)_C = O(\omega(\delta))$. If $\omega(\delta) = \delta^\alpha$ $(0 < \alpha \le 1)$ we get the usual Lipschitz classes.

The wavelet coefficients of functions of usual Lipschitz classes have been studied by several authors: Z. Ciesielski, Y. Meyer, S. Jaffard, M. Holschneider, and others.

In the present paper we study coefficient criteria for classes of functions with majorants satisfying the so-called Bari-Stechkin condition

$$\int_0^\delta \frac{\omega(t)}{t} dt + \delta \int_\delta^1 \frac{\omega(t)}{t^2} dt = O(\omega(\delta)) \qquad (\delta \to 0+).$$

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Geometrical and Numerical Analysis of Comprehensive Grid Generators

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Keywords: *Grid, curvature, metric.*

The present paper shows that each one-to-one, smooth multidimensional coordinate transformation deriving a structured grid is realized as a solution of a system of the Beltramian equations with respect to some metric specified in a physical geometry. The Beltramian equations model a multidimensional equidistribution principle in which the metric tensor is an extension of a scalar-valued weight function. Thus the metric tensor can be interpreted as a tensor-valued weight function. With this interpretation, the governing equations for generating grids in domains or on surfaces can be chosen the same (the Beltramian equations) while the grid properties can be realized through a choice of the tensor-valued weight functions or metric tensors. In order to control the grid properties, one needs to study the geometric characteristics of the metric tensors and their relations to the resulting grid behavior. This can be fulfilled with the aid of the theory of differential geometry of Riemannian manifolds.

The science of differential geometry is a powerful and helpful tool to boost the development of grid technology. In particular, many notions and characteristics of the geometry such as metric tensors, their invariants, first and second groundforms, curvatures and torsions of lines, mean and Gauss curvatures of surfaces, and Christoffell symbols are natural elements in defining grid quality measures and formulating appropriate variational and differential grid techniques in a unified manner regardless of the geometry of domains and surfaces [1, 2].

The current paper gives an account of the geometrization of the comprehensive grid formulation proposed by the author in [1] and described in detail in [2]. It also presents an important generalization of the method [1] aimed at controlling the required grid properties. The paper applies some of the relations

of the Riemannian geometry to obtaining new equations for generating grids with prescribed properties. Taking advantage of the relations established, the equations are converted into a compact form convenient for the numerical treatment by available algorithms. Studies of the behavior of the coordinate lines near boundary segments of the monitor surfaces and physical domains are carried out. Some relations of the mean curvatures of the monitor surfaces to the Beltramian equations for grid generation are exhibited. On the basis of an analysis of equations with boundary and interior layers [3] a form of the monitor functions for generating layer-resolving grids is established.

Numerical experiments of grids controlled by the geometrical characteristics of the monitor surfaces are demonstrated.

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Estimation of Curvatures from 3D Scattered Point Data

X. Li*, R. J. Cripps

Keywords: *Scattered data, surface fitting, differential geometry, curvature estimation, implicit quadratic surface, singularity.*

A key component of reverse engineering is fitting surfaces to scattered point data. The approach normally adopted is to estimate the local surface characteristics, or features, at each point by fitting a low order parametric polynomial surface to a neighborhood of nearest points ([1], [2]). Points can then be grouped into coherent sets and interpolated or approximated by surfaces corresponding to their common curvature characteristics. Clearly, the accuracy of the estimation of the geometric features has a direct effect on the quality of the finally fitted surfaces. The parametric approach has several disadvantages. Firstly the choice of parameterisation is not well defined and has an effect on the quality of the fitted surfaces [3]. Secondly, the use of low order surfaces restricts the range of shapes that can be recognized, whilst using high order surfaces requires highly complex estimation procedures [3]. Finally, least squares result in a smoothing of the information, which may not always be appropriate. Thus, for data sampled from highly shaped components, existing parametric methods do not always achieve satisfactory estimates.

In order to simplify the local fitting and to extend the range of possible surface forms that can be detected, we present an approach based on an implicit surface to estimate the local surface curvature properties. We interpolate a neighborhood of each point with a general implicit quadratic surface of the form:

$$a_1x^2 + a_2y^2 + a_3z^2 + a_4xy + a_5yz + a_6zx + a_7x + a_8y + a_9z + a_{10} = 0 \qquad (1)$$

whose coefficients are used to compute the surface curvatures. Two major difficulties arise. First is the existence of zero normal vectors, which make estimation of curvature invalid. To limit the class of surfaces to those where the normal vector is non-zero, we introduce a measure, Φ, which indicates a surface's regularity. Whilst theoretically, singular surfaces ($\Phi = 0$) can be detected, computationally we need to consider the stability of Φ as we approach singular conditions. The second difficulty is limiting the class of surface represented by (1) to a sub-class that contains surfaces that will give acceptable curvature estimates.

We present an implementation that currently avoids using least squares except when we encounter a possible singular surface. Extending the neighborhood gives an approximating surface. Φ again is used to determine a valid curvature estimate or to confirm a singular point. We further present a method for limiting the class of surfaces represented in (1).

We compare the performance of our method against the Darboux frame approach [1] by application to a sample point set generated from an analytic surface, where the true curvature values are known.

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The Stream Surface in Flow Visualization Based on Space Curve Theory

Zhenquan Li*, Gordon Mallinson

Keywords: *Space curve, stream surface, mass conservation.*

The three-dimensional visualisation for fluid flow has been attracting much attention from different areas such as computer science and engineering. Streamline visualization is an important way for helping to understand a fluid velocity

field. A stream surface in a three-dimensional fluid flow vector field is a surface across which there is no flow. Stream surfaces can be useful for flow visualization because they enable the scientist to isolate part of the flow field for detailed study; the amount of data presented in one visualization is reduced to a manageable quantity. Because no flow crosses the surfaces, restrictions of the visualized data based on stream surfaces can be more physically meaningful than various geometric restrictions. Stream surfaces can also be useful for the construction of streamlines because streamlines can be computed from the intersection of two nonparallel stream surfaces.

This paper describes a method for the calculation of stream functions in flow visualization based on space curve theory. The stream surfaces are represented by holding stream functions constant. The derived equations are given in terms of the components of the gradients of stream functions and velocity field. A simple example shows how to calculate the analytic stream function. However, the derived equations are partial differential equations that are not generally solvable analytically. Issues associated with the numerical evaluation of the stream functions are discussed in the paper.

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Smooth Trinary Subdivision of Triangle Meshes

Charles Loop

Keywords: *Subdivision surfaces, geometric modeling.*

Standard binary subdivision operators may generated surfaces with unbounded curvatures at points corresponding to extraordinary vertices. This defect can be removed by manipulating the eigenvalues of the subdivision operator to impose a bounded curvature spectrum. This procedure may lead to inconsistencies in position of new edge points, requiring an ad-hoc averaging step to restore consistency. In the trinary subdivision setting, where edges of the mesh are split 3 to 1, no such inconsistencies arise. In this talk, we generalize trinary subivision of C^2 quartic box splines to arbitrary triangulations. We argue that generalized trinary subdivision is well suited to spectral mask construction and would be difficult to contemplate without it.

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Sobolev Spaces and Native Spaces

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Keywords: *Sobolev space,native space, radial basis function.*

It's well known that the solutions of many differential equations lie in the Sobolev spaces. The working group of Goettingen has proven that the Sobolev spaces are native spaces which are the Hilbert space completion of the set of the linear combinations of the translates of a radial basis function. The Sobolev spaces are induced by a kind of radial basis functions, called Sobolev functions. It follows that the solutions of the differential equations can be approximated by linear combinations of the translates of the Sobolev functions. However, the Sobolev functions consist of modified Bessel functions which are not easy to calculate. In particular, the modified Bessel functions have singularities and are quite unpleasant. The author finds that the current Sobolev functions can be replaced by a kind of functions, which are very easy for calculation and have no singularities. In fact, these substitutes consist of the exponential functions and polynomials only.

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A Unified Framework for Cubics and Cycloids

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Keywords: *Parametric cubic curves, trigonometric curves, shape preserving representations.*

We consider the 6-dimensional space containing cubic polynomials and trigonometric polynomials of degree 1 for curve design. We describe the optimal shape preserving basis and obtain the control polygons of simple curves like complete circles and ellipses, complete cycloidal arcs, helices and sinusoids. Practical aspects of curve evaluation are included.

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Sparse Geometrical Image Representations with Bandelets

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Keywords: *Approximation, images, bandelets, geometry, wavelets.*

Sparse representations are at the core of efficient signal processing applications such as compression, restoration or pattern recognition. For multidimensional signals having some geometrical regularity, separable bases including wavelet bases provide suboptimal approximation schemes. Bandelet orthogonal bases are constructed to follow the geometrical structures of images. They provide optimal non-linear approximations of piece-wise regular images. Applications to compression and restoration will be shown.

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Local Analysis of Polyhedral Surfaces

Jean-Louis Maltret*, Marc Daniel

Keywords: *Discrete curvature, Gaussian map, polyhedral surface, triangulations, vertex analysis.*

This work aims at providing second order estimators relevant for discrete objects. The latters are defined by a cloud of points and an associated structure generally assumed to be a triangulation. Discrete Gaussian curvature is currently the most often proposed estimator. It is defined by analogy to the continous notion of Gaussian curvature. Nevertheless, a precise study states that even if this estimator is valuable, different inadequacies exist.

One must then pay a particular attention to the existing links between the discrete curvatures and Gaussian map. The goal is to set up a vertex classification of a polyhedral surface in $\mathbb{R}^3$ making a local analysis of its vertices possible. This analysis is necessary to take into account the numerous and complex situations which have no equivalent on G^2 surfaces.

A discrete Gaussian map is investigated. It is a similar tool that those yielding the classical notions of curvatures in differential geometry. One must in particular examine the vertices which are neither convex nor concave and provide relevant estimators in such cases. We restrict our study to what we can consider as the usual cases in the context of geometric modeling. It means we assume that a projection plane, called feasible plane, exists for these vertices. These notions provide a way for obtaining a feasible normal vector, which is always difficult to be defined for a vertex.

A first step of this work is achieved and proposes a qualitative and exhaustive classification of the vertices based on the spatial position of their neighbouring points.

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Polynomial Precision Clough-Tocher Interpolants

Stephen Mann

Keywords: *Triangular Bézier patches, scattered data interpolation, polynomial precision.*

One problem in scattered data fitting is the following: Given a triangulated set of points in the plane, with height values and derivatives at the points, find a smooth function that interpolates the values and derivatives at the data points. Here smooth means for the surface to be C^k, with $k \geq 0$. In this talk, I will investigate piecewise polynomial schemes for interpolating such data.

When fitting a single polynomial patch per triangle, we either have to construct a patch of relatively high degree (degree $4k+1$ for C^k continuity [7]) or we need to solve the vertex consistency problem [5]. An alternative to these two choices is to fit more than one patch per face, as done by Clough-Tocher [2], Powell-Sabin [6], and Morgan-Scott [4]. Related to the work I will present here is the Clough-Tocher work of Alfeld and Schumaker [1].

Typically, triangular interpolation schemes interpolate position and some number of derivatives at the corners of the triangles, and they also interpolate additional derivatives at the boundaries of the triangles. In earlier work, I presented an alternative to requiring derivative information on the boundaries [3]. Instead, enough derivatives were placed at the corners of a triangle to completely specify an interpolating patch; if degree n patches are used, then the resulting patches reproduce degree n polynomials. However, if used to construct a patch network, adjacent patches will only meet with C^0 continuity. So as a second step I adjusted the control points of these patches to increase the continuity between the patches. The continuity adjustment is made in such a way as to retain polynomial precision.

In this talk, I will use my continuity adjustment ideas to build Clough-Tocher interpolants, which fits three polynomial patches per triangle. This split domain scheme (as is typical of split domain schemes) achieve C^k continuity at a lower degree than my single patch per triangle schemes.

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Efficient and Accurate Computations with Algebraic Primitives for Geometric Applications

Dinesh Manocha

Keywords: *Accurate computation, boundary evaluation, medial axis computation, robustness, intersections.*

Many geometric applications like boundary evaluation in solid modeling, proximity queries and robot motion planning involve dealing with numeric data corresponding to high degree algebraic numbers. They come up in computing generalized Voronoi diagrams of lines and planes, medial axis of a polyhedron and geometric computation on non-linear primitives described using algebraic functions. Earlier algorithms dealing with algebraic primitives either use fixed precision arithmetic or techniques from symbolic computation. While the former can be inaccurate, the latter is considered too slow in practice. We present efficient representations and algorithms for reliable computations with algebraic numbers. We use these representations to efficiently perform geometric queries like inside/outside tests, which-side or orientation tests as well as solving univariate and multi-variate polynomial systems. The overall approach combines different techniques from symbolic computation based on exact arithmetic with floating point arithmetic. These include algebraic curve classification, multivariate Sturm sequences, and multi-polynomial resultants. We demonstrate its applications to efficient and reliable computation of curve and surface intersections, boundary evaluation and medial axis computations. In practice, it is more than two orders of magnitude faster as compared to earlier implementations that produced reliable results. Some of the algorithms have been implemented as part of two public domain packages, MAPC and PRECISE, available from our internet site:

`http://www.cs.unc.edu/ geom/geom_solid/index.shtml`

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Surface Completion of an Irregular Boundary Curve Using a Concentric Mapping

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Keywords: *Surface completion, surface parameterization, NURBS curve, NURBS surface, surface modeling.*

It is frequently necessary to complete the design of a surface from a specification of its boundary. Many techniques have been developed to solve this problem if the boundary is topologically a triangle or rectangle, and the curves are specified as NURBS. More complex boundaries generally require manual efforts to decompose the perimeter into topologically rectangular or triangular regions and entail auxilliary boundary specifications. The case of a single irregular boundary curve is not addressed by previous surface completion methods.

This paper introduces a technique for completing the surface when the boundary is specified by a simple, closed, planar, NURBS curve. The mapping produces a NURBS surface whose outer boundary is the input curve, and whose parameterization generalizes the polar parameterization of the disc. Therefore, the parameter values corresponding a point on the surface lend intuition to its location and proximity to the boundary. The paper explores further mathematical properties of the mapping in the context of geometric design.

The technique has theoretical significance as a means for parameterizing an arbitrary polygon. It may also provide an occasional alternative to the use of trimming curves. For example, it is often necessary to cap an open region of a surface with a planar section. The method of this paper can be used to fill this hole with a single tensor product piece.

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Interactive Modeling with Multiresolution Subdivision Surfaces

Ioana M. Martin

Keywords: *Model decoration, multiresolution editing, surface pasting.*

We present new subdivision-based algorithms that enable surface modeling operations that are difficult to achieve with existing techniques. We focus on two main classes of operations: model decoration and free-form multiresolution editing.

Model Decoration

Feature creation and placement are important especially during the initial stages of design when a large space of alternatives needs to be explored.

Surface pasting

Similar to cutting and pasting of images, surface cut-and-paste operations [1] enable efficient modeling of complex shapes without having to design every detail from scratch. For instance, shapes obtained by 3D scanning can be transferred onto relatively simple objects creating a complex look with minimal effort. Our main contributions include methods to extract a feature of interest from a source surface, to identify an area on the target surface where the feature should be pasted, and to efficiently establish the necessary mappings between the source and target surfaces for pasting at interactive rates.

Editing with sharp features

Recursive subdivision methods generate mesh hierarchies that converge to smooth limit surfaces. Many real objects, however, are only piecewise smooth and exhibit creases and sharp corners. The novel aspects of our research in this area [2] include:

(a) an algorithm for creating sharp features along arbitrary curves on a multiresolution surface without remeshing, (b) an extended set of rules for the Catmull-Clark subdivision scheme for the creation of creases along quad diagonals, and (c) a unified framework for offsetting and trimming operations.

Free-Form Multiresolution Editing

Another common design paradigm is to allow designers to interactively deform an initial geometric shape to obtain a new one that satisfies certain constraints. When the input model has high-frequency geometric detail, it needs to be preserved during global deformations of its shape. We developed a new variational approach in which deformations applied to an initial shape are viewed as a vector field over the input model. Multiresolution details are included as part of the rest shape and are preserved throughout the optimization process. We consider point, curve, and normal constraints which can be set at different resolution levels.

Acknowledgments

The research pertaining to model decoration using sharp features and surface pasting was jointly performed with Henning Biermann (NYU), Denis Zorin (NYU), and Fausto Bernardini (IBM). The variational design aspects were investigated together with Remi Ronfard (IBM, visiting) and Fausto Bernardini.

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A Link between Statistics and Approximation Theory

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Keywords: *Ill-posed problems of estimating a function, model selection.*

In statistics, there exists two classical approaches to deal with the ill-posed problem of estimating a function (like a density or a regression function) on the basis on n independent observations say. These approaches differ from the type of prior information which is known (or assumed to be known) on the target function. The parametric approach consists in assuming that the target belongs to some model defined by a finite given number of parameters. The most popular estimating procedure is then the maximum likelihood. The usual theory on these estimators is asymptotic, i.e. the number of parameters is fixed and n goes to infinity. Of course if the model is too far from reality, this approach can fail dramatically. One can prevent oneself from modeling errors by performing a data-driven choice of a model among a given list instead of taking a single model. Model selection criteria for parametric models have been developped in the early seventies. Typically, one considers (like in Alkaike's criterion for instance) the log-likelihood with a penalty term which is proportionnal to the number of parameters of the model, the justification for using such penalized criteria being purely asymptotic. The other main classical approach is said to be non parametric. Instead of assuming that the target belongs to some given

parametric model, one assumes some prior knowledge on the "variability" of the target usually expressed in terms of a control of a modulus of smoothness. Adaptive non parametric methods which have been intensively developped since the eighties propose various algorithms which all tend to make a data driven trade off between the "bias" of the non parametric modeling and the stochastic error. The purpose of this talk is to give an account of what is the non asymptotic view for model selection. This point of view allows to interprete the usual model selection criteria differently and also to correct them by taking into account not only the dimensions of the models but also the complexity of the list of models which is considered. This also allows to relax the assumption that the dimension of the models should be fixed while n goes to infinity. Hence one can consider huge lists of models with variable dimensions which are known for their good approximation properties. This leads to estimation procedures which are relevent from an adaptive non parametric view point.

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Denoising Signals Observed on a Random Design

V. Maxim

Keywords: *Wavelets, equidistant design, random design, log-spline, shrinkage.*

Wavelet shrinkage is a technique for nonparametric function estimation which has asymptotic near-optimal properties over a wide class of functions when applied on equispaced samples. In [1] authors propose a wavelet shrinkage procedure for nonequispaced samples, which is adaptative and near optimal for functions in piecewise Hőlder spaces. However this procedure concerns deterministic designs.

We propose a method to estimate functions in Hőlder classes when the design is nondeterministic. We use a log-spline model ([2]) to estimate the unknown density function which generates the design. The log-spline estimator exists except on an event whose probability tends to zero when the size of the sample grows to infinity. When this estimator exists, we show that the wavelet estimate of the unknown function is within a logarithmic factor of the global minimax risk over a wide range of Hőlder classes.

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A Recursive Computation of Tensor Product Hermite Spline Interpolants

A. Mazroui*, D. Sbibih, A. Tijini

Keywords: *Tensor product, Hermite interpolants, decomposition.*

Let $\Delta_n = \{a = x_0 < x_1 < \cdots < x_n = b\}$ and $\Delta_m = \{c = y_0 < y_1 < \cdots < y_m = d\}$ be two arbitrary partitions of $I = [a, b]$ and $J = [c, d]$ respectively, which define a rectangular partition of $\Omega = I \times J$ given by

$$\Delta_{n,m} = \{R_{i,j} = [x_i, x_{i+1}] \times [y_j, y_{j+1}],\ 0 \le i \le n \ \text{ and } \ 0 \le j \le m\}.$$

For $(k, l) \in \mathbb{N}^2$, we denote

$$C^{k,l}(\Omega) = \{g : \Omega \longrightarrow \mathbb{R} \ :\ g(\cdot, y) \in C^k(I) \ \text{ and } \ g(x, \cdot) \in C^l(J)\},$$

and $P_{2k+1,2l+1} = P_{2k+1} \otimes P_{2l+1}$ the space of bivariate polynomials of degree $\le k$ in the first variable and of degree $\le l$ in the second variable.

In this talk, we are interested in the following Hermite problem : for a given function f defined on Ω, construct a piecewise polynomial function $f_{k,l} \in C^{k,l}(\Omega)$ such that $f_{k,l|R_{i,j}} \in P_{2k+1,2l+1}$, and satisfies the following interpolation conditions :

$$D_x^p f_{k,l}(x_i, y_j) = D_x^p f(x_i, y_j), \quad 0 \le p \le k,$$

$$D_y^q f_{k,l}(x_i, y_j) = D_y^q f(x_i, y_j), \quad 0 \le q \le l.$$

The construction of $f_{k,l}$ is done recursively. More specifically, we show that $f_{k,l}$ is the sum of $f_{k-1,l-1}$ and of other particular splines which are easy to compute. We prove that this method has several advantages in comparison with the classical one. We illustrate our results by numerical examples.

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Texture Scale and Image Segmentation Using Wavelet Filters

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Keywords: *Multiscale analysis, texture segmentation, wavelet filter, Karhunen-Loeve transform.*

This paper describes a new unsupervised segmentation algorithm for textured images when the number of textures in the image is a priori unknown. The basic idea of the algorithm is the definition of a characteristic scale for a given texture. This caracteristic scale is determined using Karhunen-Loeve transforms of wavelet coefficients of texture samples: a notion of stability of features with texture sample size is then introduced, which allows to define texture scale. This analysis uses wavelet frames associated to orthogonal and biorthogonal filters, and we pay particular attention to properties of symmetry for the filters. The end of the article is devoted to the description of the segmentation algorithm, and some illustrative experiments on natural textured and real images are presented.

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A Converse Theorem for Approximation by Gaussian Networks

H. N. Mhaskar

Keywords: *Radial basis functions, Gaussian networks, weighted approximation.*

Let $s \geq 1$ be an integer. A Gaussian network is a function of the form $g(\mathbf{x}) = \sum_{k=1}^{N} a_k \exp(-\|\mathbf{x} - \mathbf{x}_k\|^2)$, where the input $\mathbf{x}$ and the *centers* $\mathbf{x}_k$ belong to $\mathbb{R}^s$, and a_k are real numbers. The *minimal separation* between the centers, defined by $q(g) := \min_{1\leq j\neq k\leq N} \|\mathbf{x}_k - \mathbf{x}_j\|$ plays a crucial role in determining the degree of approximation by interpolatory Gaussian networks, as well as the stability of the computation. We write $Q(g) := \max_{1\leq k\leq N} \|\mathbf{x}_k\|$. Let $\beta > 0$, $1 \leq p \leq \infty$, and $f \in L^p(\mathbb{R}^s)$. We prove that the following statements (a) and (b) are equivalent. (a) For all $m \geq 1$, there exists a Gaussian network g_m with $q(g_m) \geq m^{-1}$ and $Q(g_m) \leq cm$ such that $\|f - g_m\|_{p,\mathbb{R}^s} = \mathcal{O}(m^{-\beta})$, (b) For every $m \geq 1$, there exists a polynomial P_m of coordinatewise degree at most m^2 such that $\|f - P_m \exp(-\|\cdot\|^2/2)\|_{p,\mathbb{R}^s} = \mathcal{O}(m^{-\beta})$.

The relationship between weighted polynomial approximation of f as in (b), and the smoothness of f has been well understood due to the research by many mathematicians in the past 30 years. We observe that the approximation in part (a) may be nonlinear, with the locations as well as the number of the centers dependent on f, and there are no conditions on the coefficients of the networks. Moreover, the result holds for individual functions, rather than for universal approximation of a class of functions.

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Multiscale Evaluation of Geosatellite Data

V. Michel

Keywords: *Spherical wavelets, multiscale methods, regularization, inverse problems, satellite data.*

The three European satellites CHAMP (launched in 2000), GRACE (planned launch in 2002), and GOCE (planned launch in 2005) will offer data of the gravitational field (and, additionally, the magnetic field in case of CHAMP) of the Earth in an accuracy which has — from the global point of view — not been achieved before. Their evaluation yields a variety of new information for geoprospecting, oceanography, climatology, and other sciences. The available data at the satellite orbit will be given as scalars, vectors, and tensors, respectively. From the mathematical point of view all relevant problems may be represented by operator equations in separable Hilbert spaces with well-known spectral representation of the operator. In every case the downward-continuation, i.e. the mapping of the orbit data to the surface of the Earth, is exponentially ill-posed, since the singular values of the corresponding operators exponentially converge to 0 (as the degree of the orthonormal polynomials tends to infinity). Therefore, appropriate regularization techniques are required. In the talk a general multiscale regularization technique is developed for operator equations of the type described above. It is possible to derive characteristic properties of wavelets and scaling functions, such as multiresolution, approximate identity, scale step property, and reconstruction formulae, by simply using the Hilbert space axioms. The applicability of this generalized approach is demonstrated for the case

of the reconstruction of the (harmonic) density distribution of the Earth out of CHAMP, GRACE, and GOCE data. A numerical algorithm (pyramid scheme) and a multiscale denoising technique will be discussed.

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Exploiting Matrix Structure in Curve Intersection Problems

G. Casciola, F. Fabbri, L.B. Montefusco*

Keywords: *Structured matrices, Bezout resultants, curve intersection.*

In this talk we consider a classical problem in CAGD: the computation of the intersection points of two planar rational Bézier curves. This problem has been extensively studied in literature and its solution is essentially based on an algebraic or geometric approach (see e.g. [3], [4]). A comparative performance evaluation has shown that the methods based on implicitization are faster than other intersection algorithms for curves of degree up to four, but their performance degrades for higher degree curves [5]. This can be explained by recalling that such methods are based on the symbolic expansion of the Bezout determinant, and on the evaluation of the roots of the resulting polynomials, and that the solution of both these problems is less efficiently solved as the curve degree increases. In addition, for high degree curves, the implicitization based approach is strongly influenced by increased numerical problems, such that no useful results have yet been obtained for curves of degree greater than five.

In this talk we exploit the structure implicitly enclosed in the matrices involved in the algebraic approach, in order to improve the classical implicitization method, making it suitable for practical use in a real Geometric Modeling System. More precisely, we formulate the intersection problem using a Bezout matrix with polynomial entries and we evaluate its numeric-symbolic triangular factorization by means of a generalization to the polynomial ring of the fast fraction-free algorithm proposed in [1] to factorize a Bezout matrix with integer entries. This algorithm, by using the displacement structure of Bezout matrices and the relation existing between these matrices and the Euclidean scheme, succeeds in obtaining a computational complexity of $O(n^2)$.

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Inverse Spherical Surfaces with Applications to Geometric Modelling

G. Casciola, S. Morigi*

Keywords: *Spherical geometric modelling, rational Bézier patches, spherical triangle, rational geometric continuity.*

The problem of defining curves on curves, or surfaces on surfaces, plays an important role in Computer Aided Geometric Design (CAGD), in particular, the problem of defining curves and surfaces over the sphere is of a certain interest, since it allows us to model circular/spherical phenomena in a more natural way. In this talk we describe a new way to design rational parametric surfaces defined on spherical triangles which are useful for modelling in a spherical environment. These surfaces can be seen as single-valued functions in spherical coordinates. We also present some examples showing the use of these patches for modelling spherical surfaces.

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Computation for Curved Objects Using Subdivision

G. Morin

Keywords: *Exact computation, curved object, subdivision.*

We propose efficient and robust algorithms for curved objects, based on their subdivision representation. Generalizing computational geometry tools to curved objects is a challenging task, leading in some cases, to solving algebraic equations with no exact solutions. In this work, we propose to consider curved object defined by subdivision, in particular, B-splines or circular objects. The subdivision representation is multilevel and therefore allows to develop efficient algorithms using filters. Moreover, the convex hull property of the control polygon provides the possibility to guarantee the result to a given precision.

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Stable Spline Wavelets on Nonuniform Knots

Andreas Lorange, Knut Mørken*

Keywords: *Splines, wavelets, stability, non-uniform knot spacing, uniform norm.*

Classical wavelets are constructed on non-uniform grids covering finite or infinite intervals, with the stability measured in the L^2-norm. There exist a few constructions of spline wavelets on general, non-uniform knots, but the stability of these constructions is uncertain, except in the simplest cases (the constant and linear cases). In this talk we will introduce an alternative construction of spline wavelets on non-uniform knot vectors, which is stable with respect to the L^∞-norm, at least up to degree 2. Whether or not the construction can be generalised stably to higher degrees is, as yet, not known.

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Two Approaches for Solving Pseudodifferential Equations on Spheres using Spherical Radial Basis Functions

Tanya M. Morton

Keywords: *Sphere, radial basis functions, pseudodifferential equations, PDE.*

Symmetric and unsymmetric collocation using spherical radial basis functions are considered. The unsymmetric collocation approach is sometimes referred to as Kansa's method. Kansa's method has received considerable attention in recent years for $\mathbb{R}^m$, but this has not been the case for the analogous problem on the sphere. Error bounds and numerical results for Poissons equation on the sphere are presented in this paper and the advantages and disadvantages of the two approaches are discussed.

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Approximation of the Curvatures of a Smooth Surface

J-M. Morvan

Keywords: *Surfaces, meshes, curvatures, approximations, normal cycle, differential geometry.*

In classical differential geometry, the notion of intrinsic and extrinsic curvatures of a hypersurface is a major concept. At each point, they are related to the Riemannian curvature tensor, and the second fundamental form of the immersion. It is possible to get global informations by integrating these quantities over the hypersurface itself, or over its subdomains. An important question consists in generalizing these quantities to subsets of an Euclidean space without differentiability conditions. We shall give a summary of the subject, and apply this theory in the case of polyhedral surfaces, in particular triangulations. Moreover, we shall give approximation and convergence results, when a smooth surface is approximated by a triangulation inscribed on it.

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Algebraic Methods for Implicit Curves and Surfaces

B. Mourrain

Keywords: *Implicit equations, curves, surfaces, algebraic numbers, solving polynomial equations.*

The interaction between algebra and geometry is a long story. It involves the manipulation of algebraic numbers or the resolution of polynomial equations, which is particularly crucial nowadays in many applications, going from CAD to Computer vision. In this talk, we will give a brief overview of different approaches for tackling this question, including homotopic, subdvision, algebraic and geometric methods. These will be illustrated in problems on (algebraic) curves and surfaces.

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Classical Geometric Methods for the Computation of Minkowski Sum Boundary Surfaces

Heidrun Mühlthaler*, Helmut Pottmann

Keywords: *Minkowski sum, convolution surface, rational parameterization, ruled surface, canal surface, Laguerre geometry, general offset, offset surface.*

The Minkowski sum is a basic geometric set operation which arises in various applications such as geometric tolerancing, NC tool path generation and robot motion planning. The boundary of the Minkowski sum of two surfaces is part of the so-called convolution surface of the two input surfaces. In most cases, convolution surfaces can be computed only by numerical algorithms.

The present paper applies classical geometric methods in a study of surface pairs for which the convolution surface can be explicitly parameterized. It is shown how to compute a parameterization for the convolution surface of two ruled surfaces. Moreover, we investigate the rational convolution surface of two rational ruled surfaces and the connection to rational parameterizations of offsets of rational ruled surfaces. Using methods of Laguerre sphere geometry, the convolution surface of two canal surfaces and special pairs of rational surfaces with a rational convolution surface are investigated.

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A Practical Approach to Manipulating Topological Maps

Nguyen Dong Ha

Keywords: *Computational geometry, topological map, planar map, GIS tool, point location, edit map.*

Topological map is a fundamental structure in computational geometry. A topological map may be composed of several primitives such as vertex, edges and faces with mutual relations among them. Because of the complex structure of a topological map, the job of manipulating it, which includes query and edit spatial objects, is not easy for many developers. The problem is that when a topological map contains too many primitives with many complicated mutual relations, the cost to query and update its spatial objects and relations increases much. An edge is the central primitive in a topological map. It can be a straight line, a polyline or a curve. This paper presents our study on topological map with several kinds of edge such as line, polyline and curve in practice. We especially focus on the ability of Topological Map and Planar Map package of the computational geometry algorithms library CGAL, which is a large scaled project funded by the European Community, in solving the main problem. This approach to the hardness of manipulating a topological map is simple, robust and has good performance. As such, it seems to be the good choice for developers who are working in computational geometry fields especially GIS.

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Pairs of B-splines with Small Support on the Four Directional Mesh Generating a Partition of Unity

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Keywords: *B-splines, complete Σ_1-splines, complete Λ_1-splines, minimal degree.*

Let τ be the four directional mesh of the plane and let Σ_1 (resp. Λ_1) be the unit square (resp. the lozenge) formed by four (resp. eight) triangles of τ. We study spaces of piecewise polynomial functions of C^k in the plane with supports

Σ_1 or Λ_1 having sufficiently high degree n, which are invariant with respect to the group of symmetries of Σ_1 or Λ_1 and whose integer translates form a partition of unity. Such splines are called complete Σ_1 and Λ_1-splines. We first compute the dimension of these spaces in function of n and k. Then, for any fixed $k \geq 0$, we prove the existence of complete Σ_1 and Λ_1-splines of class C^k and minimal degree, but they are not unique. Finally, we describe algorithms allowing to compute the Bernstein-Bézier coefficients of these splines.

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Local Lagrange Interpolation by Cubic Splines on Triangulations

Günther Nürnberger

Keywords: *Bivariate splines, Lagrange interpolation, triangulations, optimal approximation order, 3D-splines.*

We describe local Lagrange interpolation methods for cubic C^1-splines on arbitrary triangulations (where about half of the triangles are subdivided by a Clough-Tocher split). The construction of interpolation points is based on an appropriate coloring of the triangulations. The interpolating splines yield optimal approximation order and can be computed with linear complexity. Similar results hold for C^1-splines on convex quadrangulations with diagonals. Moreover, we describe a local Lagrange interpolation scheme for cubic 3D-splines by using coloring methods. Numerical examples with up to 10^6 real world data show the efficiency of the methods. The results were obtained in cooperation with N. Kohlmüller, L.L. Schumaker and F. Zeilfelder.

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Smoothness of Nonlinear Subdivision Based on Median Interpolation

P. Oswald

Keywords: *Nonlinear curve subdivision, median interpolation, Hölder regularity.*

We give a refined analysis of the Hölder regularity for the limit functions arising from a nonlinear pyramid algorithm proposed by Donoho and Yu [1] for triadic refinement. Although the work in [1] is motivated by applications to robust removal of non-Gaussian noise, the Donoho-Yu scheme can be interpreted as a nonlinear subdivision process where new points are inserted based on local quadratic polynomial median interpolation.

We introduce an analogon of the Donoho-Yu method for dyadic refinement, and show that its limit functions are in C^α for some $\alpha > 1$. In the triadic case, we improve the previously known lower bound of $\alpha > \log_3(135/121) = 0.0997...$ to $\alpha > \log_3(135/53) = 0.8510....$ The improved bounds are the result of deriving recursive inequalities for second rather than first order differences, and they are close to the conjectured exact values. We also discuss some closely related nonlinear subdivision schemes. Our case study [2] hopefully fuels further interest in some open conjectures concerning nonlinear subdivision processes and their relationship with linear schemes.

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Constrained Bivariate Histosplines

P. Costantini, F. Pelosi*

Keywords: *Bivariate histosplines, volume-matching conditions, shape constraints, tension properties.*

The approximation of density functions using univariate *histosplines* (i.e. splines which satisfy the area-matching conditions arising from a given histogram) has firstly been proposed in the pioneering paper [4] and then investigated by several authors, whose efforts have been mainly devoted to avoid negative values and extraneous oscillations of the constructed function (see e.g. [1],[2],[3]).

Although equally important, the analogous bivariate problem has received much less attention and, so far, no method seems to be available for the construction of histosplines which preserve the positivity and the local monotonicity of the histogram.

In this talk we present a scheme for the construction of bivariate spline functions satisfying volume-matching conditions and subject to shape constraints (non-negativity and local monotonicity) obtained from the histogram. The basic idea of our method is to use piecewise C^1 variable degree polynomial splines with tension properties, in the sense that each polynomial patch tends, for limit values of the tension parameters, to a piecewise constant function. We discuss two possible approaches for the construction of such bivariate histosplines: the first based on a tensor-product scheme and the second, more complex but also more efficient in the shape representation, based on the Boolean sum of univariate variable degree operators.

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Reconstructing Objects with Planar Faces

M. Peternell

Keywords: *Segmentation, reconstruction, objects with planar faces, modeling, clustering, space of planar faces.*

Segmentation algorithms and techniques for modeling of objects with planar faces shall be presented. Such objects are used in several applications. For practical reasons it shall be assumed that the objects are piecewise linear over a planar domain, but not necessarily continuous. Discontinuities between the faces of the object are allowed.

At first, a segmentation technique for planar faces shall be presented which is based on a clustering algorithm in the space of planar faces (over a planar domain). At second, creating models of the objects is discussed and the algorithms are applied to the construction of building models from data point clouds.

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Enclosures of Curved Geometry and their Applications

Jörg Peters

Keywords: *Surfaces, approximation, adaptive, error control.*

An enclosure of a function, curve or surface is a two- or multi-sided approximation that tightly sandwiches the function, curve or surface. Enclosures may therefore be viewed as approximate implicitizations with known error. Such bounding constructs are useful to support, say, collision detection, re-approximation for format conversion, meshing with tolerance, or silhouette detection (see attached Figures).

This talk will survey a class of very efficient enclosures that are based on representation-specific *pre-optimization* in the max-norm and that are *predictively refinable*, i.e. the number of refinement steps can be stated *a priori* so as to enforce a maximal width of the enclosure [1,2,3].

The talk will emphasize a certain duality of optimal enclosures for uniform splines and their control polygon, show applications and explain some open problems concerning best approximation.

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Wavelet Frames and Their Applications to Wireless Transmission

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Keywords: *Wavelet frames, error correcting codes, wireless multimedia.*

Decompositions in wavelet frames give a nice opportunity to combine two conflicting properties: compression of data and their redundancy. So they look like an ideal tool for coding multimedia data (like images and video) for transmission through noisy channels. We give a short review of the theory of wavelet frames and the methods of their constructing. A few samples of the recovery of significantly corrupted images will be presented.

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Wavelets Bases on the Interval and Applications

Laura Gori, Laura Pezza*

Keywords: *Refinable functions, wavelets, multiresolution analysis on the interval.*

Starting from a particular class of refinable functions on $\mathbb{R}$, a construction of refinable functions bases on the interval is developed. Also the relative wavelets bases is constructed and some applications are provided.

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Convex Combination Maps over Triangulations, Tilings, and Tetrahedralizations

Valérie Pham-Trong

Keywords: *Convex combination map, triangulation, tiling, tetrahedralization.*

A convex combination map over a set of vertices of a two or three-dimensional mesh is a map such that the image of every interior vertex is a convex combination of the images of its neighbouring vertices. It is important that the convex combination maps used for parameterization ([3]) and morphing ([4], [5]) are one-to-one. For triangulations, injectivity is guaranteed when the image of the boundary is convex ([6],[2]). An interesting question is whether this convexity condition also guarantees injectivity over other kinds of meshes. On the one hand, we show that the condition is sufficient for tilings, by generalizing the theory of [2]. Our approach simplifies similar theory developed in [6] and [1]. On the other hand, we give a counterexample for meshes consisting of tetrahedra.

This is joint work with Michael Floater.

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A New Family of Wavelets on the Interval

F. Pitolli

Keywords: *Multiresolution analisys, optimal basis, wavelet.*

It is well known that the Multiresolution Analysis (MRA) is a powerful tool in many fields, since it produces in a natural way a multiscale decomposition of a given function. Although the MRA first originated on unbounded domains, in many numerical applications, such as image analysis or solution of operator equations, one faces the problem of approximate functions defined on bounded domains.

Restricting ourselves to the unidimensional case, the standard approach to construct wavelets on a finite interval I starts from a MRA on I generated by interior scaling functions and some adapted edge functions; those ones are especially chosen in order to satisfy useful properties, such as the polynomial exactness (see [2] and references therein). In the spline case a MRA on I, endowed with the desired polynomial exactness, can be generated in a natural way by the B-spline basis with multiple knots at the endpoints [1]; however, this construction strongly relies on the general spline theory, and can not be extended to other scaling functions.

Here, we want to follow a more general approach. We will start from a MRA on I generated by an optimal basis constructed by means of a totally positive scaling function [3]. In this construction the polynomial exactness is immediately achieved without any further adaptation. Then, we will construct the corresponding wavelet bases, and will analyze their approximation properties. Some numerical examples will be also given.

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A DCT-like Transform that Maps Integers to Integers

Gerlind Plonka

Keywords: *DCT, lossless compression, wavelet.*

The discrete cosine transform (DCT) has been extensively used for compression and transmission of signals and images. However, the cosine matrix has irrational entries. When the input data of a signal or an image consist of integers, then the transformed output generally no longer consists of integers. In many applications (e.g. JPEG), the resulting output is just rounded to integers again making the procedure irreversible. For this reason, the DCT is only used for lossy coding.

In this talk we want to present an invertible DCT-like transform that maps integers to integers. This transform is based on a new orthogonal real factorization of the DCT-II in [2]. For the new algorithm, we adapt a method for the Haar wavelet transform for integers in [1].

We present some properties of the new transform and show that it possesses the important property of the DCT to separate the frequencies of a signal. We compare it with some recent integer-to-integer wavelet transforms for lossless coding.

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A Geometric Approach to Optimization with Moving and Deformable Objects

Helmut Pottmann

Keywords: *Geometric optimization, active contours, registration, surface approximation.*

A number of optimization problems which occur in Geometric Computing involve moving or deformable objects. An important example is the so-called *registration problem* in Computer Vision; it requires the computation of an optimal motion (or similarity) which transforms two (or more) geometric models onto each other. Applications concern the transformation of a set of measurement points of an object onto a CAD model of that object or the merging of partially overlapping scans of an object. This has important applications, e.g., in 3D photography, 3D measuring, medical imaging and industrial inspection.

Another example are problems based on *active contours*, which are widely used in Computer Vision and Medical Imaging. Even the *surface approximation problem* in Geometric Design may be formulated in this way: a parametric surface or subdivision surface is deformed iteratively such that it finally approximates a given geometric model (or data set) in an optimal way. This point of view has the advantage that the u, v-surface parameters to the given data points do not have to be specified, and that we can also handle the *approximation with subdivision surfaces.*

The present talk presents a geometrically motivated optimization strategy for the solution of the problems outlined above. It is based on a *faithful local quadratic approximation of the squared distance function to a curve or surface,* and on a *linearization of the motion or deformation.* Understanding the geometry of these two local approximations and combining it with regularization techniques, we can provide efficient solutions to geometric optimization problems with moving or deformable objects. We will illustrate the method at hand of surface approximation and registration for industrial inspection.

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Radial Basis Function Interpolation on Manifolds

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Keywords: *Radial basis functions, interpolation, manifolds.*

Let values of a function $f(\underline{x})$, $\underline{x} \in \mathbb{R}^d$, be given at points that are in general positions, and let the radial basis function method provide the interpolant $s(\underline{x})$, $\underline{x} \in \mathbb{R}^d$. The calculation of s is straightforward even if d is very large, but then the approximation $s(\underline{x}) \approx f(\underline{x})$, $\underline{x} \in \mathbb{R}^d$, may be sufficiently accurate only if the number of data points is huge. In many applications, however, there are strong correlations between the variables, so the approximation is of interest just in the case $\underline{x} \in \mathcal{M}$, where $\mathcal{M}$ is a low dimensional manifold of $\mathbb{R}^d$. Thus, assuming that all the data points lie on $\mathcal{M}$, the number of independent variables of the interpolation problem is the dimension of the manifold, m say. Fortunately, the radial basis function method takes advantage of this situation automatically, without having to locate the manifold specifically. Further, numerical calculations show that the accuracy of the approximation $s(\underline{x}) \approx f(\underline{x})$, $\underline{x} \in \mathcal{M}$, depends more strongly on m than on d. These experiments are described and discussed for linear and multiquadric radial functions. In particular, the accuracy that occurs for $m=2$ and large d is compared with the usual order of accuracy in the case $d=2$.

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Manipulating 3D Implicit Surfaces by using Differential Equation Solving and Algebraic Techniques

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Keywords: *Implicit curves and surfaces, intersections, topological offsets.*

Geometric modeling by using implicit algebraic surfaces is becoming a very active research area in Computer Aided Geometric Design: in fact the simultaneous availability of the parametric and the implicit representation of a surface is extremely useful when solving central problems such as

- visualizing implicit curves in 2D space and implicit surfaces in 3D space,
- ray tracing,
- intersection problems including sectioning and offsetting, and
- extreme location, convexity area detection and closest points computations.

In this work we formulate each one of the above mentioned problems as a first order system of ordinary differential equations whose solution curve will be determined either numerically or in terms of power series whose parameter will be its arc length. Before solving the involved system of differential equations, an algebraic/topological analysis of the considered surfaces is performed in order to determine the number connected components and to detect singularities, autointersections, etc.

For surface intersection problems similar formulations to the considered here can be found in [1], [2], [3], [4] and [5].

As a by-product, these algorithms motivate the introduction of the notion of "*topological offset*" where the attention is more focused in shape than in distance: for most distances the offset of a parabola is not topologically a parabola (including the appearing of a singular point). Algorithms computing and manipulating these topological offsets will be also presented.

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Computation of Nonuniform Spline Wavelets

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Keywords: *Nonuniform spline wavelets, minimum support, semi-orthogonality.*

Nonuniform B-wavelets are polynomial spline wavelets of minimal support over nested nonuniform knot sequences. For the wavelet spaces corresponding to nested nonuniform spline spaces, the existence and uniqueness of a (semi-orthogonal) B-wavelet basis is established in the paper [1] for arbitrary polynomial degree, along with various other properties of B-wavelets. L_p-Stability issues for the piecewise linear case are considered in [2] and [3]. The aim of this presentation is to discuss several different ways to compute nonuniform B-wavelets for given knot sequences, and point out the respective advantages and disadvantages.

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Jacobi-Bernstein Basis Transformation

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Keywords: *Bernstein polynomials, least-squares approximation, orthogonal polynomials, basis transformation.*

The Jacobi polynomials $P_n^{(\alpha,\beta)}(u), \quad \alpha,\beta > -1$ are orthogonal polynomials on $[0,1]$ with respect to the weight function $w(u) = 2^{\alpha+\beta}(1-u)^\alpha u^\beta, \alpha, \beta > -1$. In this paper we follow the methodology in [1] to derive the matrix of transformation of the Jacobi polynomials into Bernstein polynomials and vice versa. We study also the stability of these linear maps of some cases and show that the Chebyshev-Bernstein basis conversion is remarkably well-conditioned, allowing one to combine the superior least-squares performance of Chebyshev polynomials with the geometrical insight of the Bernstein form. We also compare it to other basis transformations.

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Variational Interpolation on Compact Homogeneous Manifolds: the Norming Set Approach

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Keywords: *Norming sets, interpolation error, markov inequalities, radial basis function.*

Approximation theory results in the general setting of an embedded manifold $M^d \subset \mathbb{R}^{d+r}$ such that M is the orbit of some subgroup G of $O(d+r)$ the orthogonal group in $\mathbb{R}^{d+r}$ have been known since[5]. In particular Bernstein's inequality for trigonometric polynomials was generalized to such spaces and lead to many

converse type theorems as well as theorems on asymptotic widths. Eventually full proper setting for Bernstein's inequality was established in [1].

We consider interpolation on such manifolds, M, using invariant kernels, κ satisfying $\kappa(gx, gy) = \kappa(x, y)$. We apply the methodology of Dyn et. al. [2] and Jetter et. al. [3] to produce global error estimates for interpolation. Also, following a suggestion of J. Ward, we give local error estimates for interpolation on such manifolds. Such local error estimates are already available due to Levesley and Ragozin [4] and here we wish to compare the results achieved via the two different approaches.

The Bernstein inequality for the sphere is used in [3] to show the existence of a norming set for the polynomials of a fixed degree N with point separation at most $1/2N$. We shall produce both global and local error estimates for interpolation on M using the Bernstein inequality on M from [5]. Among other results we prove a Markov type inequality which allows us to infer the existence of norming sets supported on proper spherical subsets S_η of M with point separation of order $1/N^2$.

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A Multiresolution Method for Detecting Higher Order Discontinuities from Irregular Noisy Samples.

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Keywords: *Multiresolution, irregular samples, higher order discontinuity, noise.*

We consider the problem of detecting higher order discontinuities of univariate or bivariate functions that are represented by discrete noisy data sets. The starting point of our work are results presented in [1][2][3] concerning the use of wavelets to detect discontinuities of a function f from regular samples, i.e. from $y_i = f\left(\frac{i}{N}\right) + \varepsilon_i \quad i = 0, 1, ..., N$, one tries to find σ with $f(\sigma^+) \neq f(\sigma^-)$. First, we present a generalized version of Wang's theorem in order to relate a higher order discontinuity i.e. $f^{(k)}(\sigma^+) \neq f^{(k)}(\sigma^-) \quad k = 1, 2$ with a local maximum of the appropriate wavelet transform of the given exact data. Then, we describe an algorithm that allows to detect the discontinuity for the case of noisy sampling data. Furthermore, we analyze an approach to extend our method to irregular samples. Finally, we outline the analogous method for detecting discontinuities in bivariate functions.

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Curves and Surfaces on Study's Quadric

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Keywords: *Study quadric, motion interpolation, Steiner motion, Darboux motion.*

This paper studies special curves and surfaces on a special hyper-quadric in a seven dimensional projective space referred to as Study's Soma space. These curves and surfaces have special connotation since they represent motions in the Euclidean space. Studying their properties can lead to better ways for motion interpolation as well as developing methods to design and program mechanisms and robotic systems. They can also be used in key framing in computer animation and motion analysis in experimental biomechanics.

Eduard Study introduced in the appendix of his book: "*Geometrie der Dynamen*" in 1903 an algebraic apparatus to describe the general spatial displacement of a body depending on six parameters in the Euclidean three-space. This method is used by several other authors, such as Ravani, Husty, Gfrerer. The spatial displacement is defined as a combination of translation and orientation. Study called a position of a body a soma and used eight homogeneous coordinates to describe it. This description of a body is usually denoted by $SE(3)$

- a six-parametric Lie group embedded into a seven-dimensional real projective space P_7.

Each Euclidean congruence, motion (trajectory) or two parametric motion represents a point, curve (trajectory) or subsurface (a two-dimensional submanifold) in P_7 respectively. Hence these geometric objects lie on a six-dimensional hypersurface. These collineations (projective transformation) satisfy two quadratic conditions: an equation $(xy) = x_0y_0 + x_1y_1 + x_2y_2 + x_3y_3 = 0$ which defines the hyperquadric and is usually called Study's quadric S_6^2. It comes from the orthogonal condition between the real and dual part of a dual quaternion Q, and second from an inequality $(xx) = x_0^2 + x_1^2 + x_2^2 + x_3^2 \neq 0$, whereby we are going to use $(xx) = 1$, the Euler-equation, thus *dual unit quaternions*.

The Study quadric is a quadric of hyperbolic type with signature null. This quadric embeds special linear subspaces, which again represent special motions in the Euclidean three-dimensional space.

In this paper, we first study the sub-spaces of Study's quadric and work out the Euclidean representation of motions corresponding to curves and surfaces in these sub-spaces. We will then apply the results to the study of two important types of motions - Darboux motions and Steiner motions. Darboux motions generate planar point trajectories and therefore are useful in many mechanical engineering applications involving design of mechanisms and devices. Steiner motions are two parameter motions whose point trajectories are Steiner surfaces, or sometimes called Roman surfaces. These surfaces have been used in the past in CAGD (Computer Aided Geometric Design) (see, for example, Sederberg and Anderson).

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Reconstruction and Animation of Surfaces

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Keywords: *Quaternions, de Casteljau algorithm, surfaces, animation.*

Starting from some ideas published by Ge in the nineties to generate Steiner surfaces using quaternions we describe a method to not only build the surface but animate it starting from some known movements at some parts of the surface do. There are several examples in medical anatomy of this kind of surfaces where we know very well the movement of some parts (for example valves of the heart, stomach and intestines) and we want to reconstruct the movement of the whole surface of the organ. The objective of this application is to give doctors the oportunity to detect strange forms, such as tumors and coagulums, that normally changes the way the organ moves or it's natural shape, using only a few control points. This guides us to a less expensive and quicker diagnostic. The mathematical tool we use is a quaternionic version of the de Casteljau algorithm applied twice: once to build the surface and the second time to animate the control net that defines the surface. Using quaternions gives us a more detailed final structure, which means an accurate diagnostic that is vital to doctors and patients.

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On the Solubility of Fairing Problems

Ulrich Reif

Keywords: *Fairing problem, existence of solutions, Willmore surface.*

Energy functionals involving curvature data are widely accepted as criteria to judge the fairness of free form curves and surfaces. In this talk we address the question whether such functionals can be justly used to improve fairness, i.e., whether a minimum is attained within a given class of competing variations. While the problem is almost trivial in the curve case, the surface case is far from being completely understood. The results we present here ensure the existence of a minimizer of a suitably modified total curvature in an $H^{1,\infty}$-bounded neighborhood of the given surface. Area-depending penalties may be included as well as additional $H^{1,\infty}$-continuous constraints on the sought surface.

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Polysplines – A New Method in CAGD

H. Render

Keywords: *Multivariate polyspline, polyharmonic functions, cardinal interpolation.*

The theory of polysplines has appeared recently as an immediate generalization of the one-dimensional theory of splines, see the monograph *O. Kounchev, "Multivariate Polysplines. Application to Numerical and Wavelet Analysis", Academic Press, 2001.* The main concept of the polysplines is that the building pieces are polyharmonic functions of order p (for some fixed integer $p > 0$; harmonic functions are the simplest case of $p = 1$), and the "break–points" (or interface points) of the one–dimensional spline theory are replaces by "break–surfaces" (or interface surfaces) Γ_j, $j = 1, ..., N$ of dimension $n - 1$, on which the pieces join smoothly. In the *simplest* special cases one may take the surfaces Γ_j to be concentric spheres, or parallel hyperplanes; in dimension $n = 2$ these are concentric circles and parallel lines respectively. Experiments with data from CAGD are available in Chapter 6 of the above book, also downloadable at www.math.bas.bg/kounchev.

It is a remarkable fact that one may introduce in a unique way the theory of *cardinal polysplines* – this may be done in at least two geometrical configurations – by taking infinitely many concentric spheres Γ_j (of radii $r_j = ab^j$ for some fixed constants $a, b > 0$) or by taking infinitely many equidistant parallel hyperplanes Γ_j. It is possible to prove non–trivial generalizations of the classical cardinal interpolation theorems of Schoenberg: If some data functions f_j are given on Γ_j and if they satisfy some growth conditions with j, then it is possible to interpolate them through polypslines of order p. The precise formulation and the proofs are available in the papers of the authors, also downloadable at http://www.uni-duisburg.de/FB11/STAFF/Render/Render_home.html

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Recursive Connectivity Encoding for Mesh Stripification

Ioannis Ivrissimtzis, Christian Rössl*, Hans-Peter Seidel

Keywords: *Mesh compression, connectivity encoding, mesh stripification.*

With triangle meshes having emerged as the de facto standard for the representation of 3D models mesh compression has become an important area of research. One particular problem is the encoding of mesh connectivity besides the coding of geometry and surface attributes. We recently proposed a divide-and-conquer algorithm for encoding arbitrary triangle meshes. This algorithm is efficient and extremely simple due to its recursive nature.

We will show that this technique can still be improved and that it can easily be extended and employed for other task. A particular application we will focus on is the stripification: the mesh is decomposed into triangle strips for fast rendering. A solution to this problem can be constructed based on the results of our encoding algorithm.

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A de Boor Type Algorithm for Tension Splines

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Keywords: *Chebyshev systems, hyperbolic splines, knot insertion.*

We consider a special knot insertion algorithm for splines which are piecewisely in the linear space spanned by $\{1, x, \mathrm{Sinh}(px), \mathrm{Cosh}(px)\}$, *i.e.* tension splines with uniform tension $p > 0$. Despite its unusual shape preserving and approximation properties widely used in removing extraneous inflexions and solution of singularly perturbed boundary value problems, little is said about the numerical representation. We show how existing direct and recursive algorithms use dangerous subtractions and can therefore be unstable, especially for calculating the B-spline basis. One possible remedy is to evaluate tension spline by inserting the evaluation point in the knot sequence with maximal multiplicity. We use facts from the theory of Chebyshev splines and treat tension splines as Chebyshev ones. However, various product representations of the associated differential operator $D^2(D^2 - p^2)$ exist, and we choose one that leads to hyperbolic splines in the first reduced Chebyshev system. An analysis follows, showing that certain combinations of hyperbolic functions can be evaluated to the machine precision. For $p = 0$ we obtain cubic splines and the well known de Boor's algorithm, but the linear case $p \to \infty$ is also covered by the instance of the same algorithm. The knot insertion matrices involved behave nicely with respect to the change in tension parameter within range $0 < p < \infty$, but also with respect to the knot sequence. Thus, we can also raise a multiplicity by the knot insertion, and the implementation of the algorithm can use de Boor's maxime 'anything times zero is zero'. Numerical experiments illustrate the theory.

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Rational Interpolants with Tension Parameters

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Keywords: *Rational interpolation, tension parameters, shape control.*

Rational interpolation is a topic of great interest in Computer Aided Geometric Design, but literature is extremely poor in this field because very few authors have tried to face all the difficulties it arises. Moreover the solutions proposed up to now haven't turned out to be very useful in applications because computationally heavy, little flexible for geometric modelling purposes and most of all inadequate to be implemented in a NURBS based CAD system. These considerations motivated the research reported in this talk, where we propose a NURBS version of the parametric/non-parametric rational cubic spline interpolants with tension given in [3] and developed to improve classical polynomial tension methods in the univariate case.

Successively, the rational class proposed in [3] was extended by Sarfraz to design an interpolant parametric surface of a given rectangular set of points [5]. His work seems to be the unique proposal known in literature to solve rational interpolation problems with tension in the bivariate case, but it presents a scarce flexibility in geometric modelling systems, suffers from a high computational complexity and achives a nearly C^2 continuity only. These considerations motivated us to analyse alternative proposals of rational parametric/non-parametric surfaces which interpolate a given rectangular set of points, resolve the drawbacks of Sarfraz's method and can be interactively modified by changing the tension parameters with a resulting local/global shape control. In order to be useful in a geometric modelling system the proposed interpolation surfaces have been turned into a NURBS representation. The approach presented has been implemented and tested in our NURBS based *xcmodel* geometric modelling system.

The next step will be the generalization of this rational interpolant class to a set of unorganized points.

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Generalized Shift-Invariant Spaces

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Keywords: *Shift-invariant spaces, wavelets, frames, fiberization.*

A shift-invariant (SI) system in $L_2(R^d)$ is a collection of some functions together with all their lattice translates, for some lattice L. A generalized shift-invariant (GSI) system is a countable union of SI systems, where we allow the lattice L to vary from one SI system to another. A wavelet system (associated with a general, not necessarily integer, dilation) is, thus, not SI but is always GSI.

We study two properties of GSI systems: the Bessel property and the frame property. The characterization is done via the norm and the inverse norm of certain finite order matrices. We also use those matrices in order to study two different oversampling techniques of GSI systems: uniform and oblique.

For the case of a tight frames, all the associated matrices are equal to the identity map, and the characterization is then reduced to scalar identities. This induced characterization of tight frames was obtained independently by Hernandez and Weiss (by other techniques).

A wealth of special cases are covered by this uniform framework. Those include SI systems, wavelet systems and many variations of the wavelet theme.

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Efficient Sampling in Dynamic Tomography

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Keywords: *Sampling on lattices, Shannon theory, Radon and X-ray transforms, helical tomography.*

In 2D tomography we want to reconstruct a function f from its line integrals, i.e. from its Radon transform

$$Rf : g(\phi, s) = (Rf)(\phi, s) = \int_{-\infty}^{+\infty} f(s\theta(\phi) + u\zeta(\phi))\, du$$

where $s \in \mathbb{R}$, $\phi \in [0, 2\pi[$, $\theta(\phi) = (\cos\phi, \sin\phi)^t$ and $\zeta(\phi) = (-\sin\phi, \cos\phi)^t$. Medical (or industrial) scanners sample the Radon transform through the measurement of the X-ray attenuation on a finite number of lines from source to detector positions. Thus a crucial practical question is the sampling condition of the data g for the reconstruction of f. The Fourier analysis is a powerful tool to handle this question. We must first identify the essential support K of the Fourier transform of the function to be sampled (here g). Then we can propose a regular sampling scheme generated by a non-singular matrix W (g is sampled at $g(Wl), l \in \mathbb{Z}^2$), such that the sets $K + 2\pi W^{-t}k, k \in \mathbb{Z}^2$ do not overlap (Shannon condition). Among the schemes satisfying the Shannon condition, efficient sampling schemes maximize $|\det(W)|$, i.e., the fundamental area of the scheme, thus minimize the number of sampling points to identify g. In tomography, it has been shown by Rattey and Lindgren [3] that the essential support K of the Fourier transform of the Radon transform of an essentially band limited function has a bow tie shape. From this result, the interlaced scheme is shown to be optimal in 2D tomography. The mathematics of this analysis can be found in [2].

In this work we aim to explore sampling conditions of dynamic 2D tomography. The motivation for this study is cardiac tomography. In this case the scanner is rotating around the patient whereas the heart is beating. The situation is very similar to 3D helical tomography where the scanner is rotating during the translation of the patient table (axis of the helix). In cardiac tomography, the table is fixed but the reconstructed function $f(x,t)$ is a function of both space ($x \in \mathbb{R}^2$) and time ($t \in \mathbb{R}$) thus cardiac imaging can be interpreted as helical tomography along the time axis. We have given in [1] the sampling conditions of the 3D X-ray transform for a circular trajectory of the detector, i.e., sampling of $g(\phi, s, t) = \int_{-\infty}^{+\infty} f(s\theta(\phi) + u\zeta(\phi) + te_3)du$, where $e_3 = (0,0,1)^t$ is orthogonal to the (θ, ζ) plane. We show that the study of helical tomography sampling schemes can be reduced to constrained sampling of the 3D X-ray transform. We give sampling conditions and sampling schemes for helical tomography that can be applied to dynamic tomography. Further generalizations to time periodic functions (such as a beating heart can be supposed to be) must be performed in order to produce sampling schemes exploiting this property.

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A Geometric Evolution Perspective for Subdivision and Surface Modeling

M. Rumpf

Keywords: *Subdivision, partial differential equations, geometric evolution problems.*

A new approach to subdivision and surface modeling based on geometric evolution problems is presented. The evolution by mean curvature motion can be understood as a natural geometric filter process where time corresponds to the filter width. Thus, subdivision can be interpreted as the application of such a geometric filter on an initial coarse surface.Hence, the method incorporates a new approach for the theoretical treatment of subdivision. The concrete scheme is a model of such a filtering based on a successively improved spatial finite element approximation starting with some initial coarse mesh and leading to a smooth limit surface.Taking into account additional forces in the evolution allows a flexible modeling of the surface under consideration. The approach comes along with a unified frame for subdivision and surface fairing concerning the geometric

foundation. It can be regarded as a cascadic multigrid methods with respect to the actual numerical procedure. The derived method does not distinguish between different valences of nodes nor between different mesh refinement types.

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Adaptive Grid Methods for Image Defined Domains

M. Rumpf

Keywords: *Adaptative grid methods, 3D image data, image processing, visualization.*

A efficient approach to describe complicated domains given by 3D image data is described. The description is implicit, hence domains are described via level set boundaries. A level set corresponds to a specific level of a piecewise multilinear finite element function on an adaptive grid. The adaptive grid is not stored explicitly in some graph data structure, but it is itself described by a error indicator value on the set of nodes. Suppose these error indicator values fulfill a natural saturation condition the adaptive grids is characterized by some nice properties, such as restricted types of spatial transitions in the grid level. The potential for image processing and visualization applications as well as simulation especially in medizine will be outlined. The efficiency of this approach with respect to cache optimality, multi grid capabilities, and implementational simplicity will be described.

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The Analysis and Control of Artifacts in Subdivision Surfaces

M. Sabin*, L. Barthe

Keywords: *Subdivision, analysis, artifacts.*

An artifact in a subdivision surface is an unwanted feature which cannot be removed purely by moving the control points at the current level of detail. Clearly as subdivision surfaces become more widely used, these need to be minimised either by careful tuning of the schemes themselves, or by careful usage, or both. For both of these we need to be able to analyse a scheme to determine to what extent it can be tuned to minimise artifacts and also how it can be used not to invoke them.

Our understanding of artifact mechanisms is currently at much the same level as our understanding of smoothness was twenty years ago. There are several different kinds, some of which are related to known concepts.

This presentation shows some results both in analysis and in control, hoping to stimulate progress, so that in twenty years time we may understand artifacts as well as we now understand smoothness.

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Algorithms for Tensor Products of C^1 Merrien Subdivision Schemes

Paul Sablonnière

Keywords: *Tensor product surface, subdivision scheme.*

We define and study subdivision and corner-cutting algorithms for surfaces defined as tensor products of univariate C^1 Hermite subdivision schemes introduced by J.L. Merrien (Numer. Algorithms, 1992). Then we give necessary and/or sufficient conditions for the bimonotonicity (i.e. monotonicity in both directions Ox and Oy) or convexity of these surfaces in terms of geometric properties of the associated control nets. An application to the construction of bimonotonicity preserving interpolants is presented in the poster of F. Foucher.

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Discretization of Certain Curves and Surfaces via Minimization of Energy or Lebesgue Constants

E. B. Saff

Keywords: *Bivariate polynomial interpolation, minimization of energy, minimization of Lebesgue constant.*

Let $K \subset \mathbb{R}^d$ be compact and $s > 0$. One method to discretize K into N points $\{\mathbf{x}^*_{i,N}\}_1^N$ is via the *s-energy*; that is, for any N points $\omega_N = \{\mathbf{x}_i\}_1^N \subset K$ we associate the energy

$$E(\omega_N, s) = \sum_{1 \le i < j \le N} \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|^s},$$

and then choose $\omega_N^* = \{\mathbf{x}_{i,N}^*\}_1^N$ so as to minimize this energy:

$$E(\omega_N^*, s) = \min_{\omega_N \subset K} E(\omega_N, s).$$

For $s < d$, the asymptotic distribution of such optimal points follows from potential theoretic methods. Here we discuss the case when $s \geq d$, for which the aforementioned methods do not apply.

Another type of discretization results from minimizing the Lebesgue constants for polynomial interpolation in d-variables. We let $N = N(n, d, K)$ denote the dimension of the space $\Pi_n(K)$ of polynomials of degree at most n restricted to K, and for any set of N points $\omega_N = \{\mathbf{x}_{i,n}\}_1^N \subset K$ for which the interpolation problem is solvable, we associate the norm of the corresponding polynomial interpolation operator $\mathcal{L}_n : C(K) \to \Pi_n(K)$, which is known as the *Lebesgue constant*. We say that $\{\mathbf{x}_{i,n}^*\}_1^N$ is optimal for polynomial interpolation if it minimizes this Lebesgue constant over all $\omega_N \subset K$. The asymptotic distribution (as $n \to \infty$) of such optimal (or nearly optimal) points is a difficult problem and here we discuss how this limit distribution (called an AIM) can be obtained for bivariate polynomial interpolation on curves of genus 0 in $\mathbb{R}^2$. Knowledge of the AIM in these cases leads to computationally simple construction for "good" interpolation schemes.

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Constructing Multiresolutions from Subdivisions : Local Techniques Using Masks

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Keywords: *Subdivision, multiresolution, biorthogonal.*

In recent works [1,2,3,4] we have developed a constructive approach for building biorthogonal multiresolutions from existing subdivisions. The approach is based upon the local least squares approximation of each point of a coarse mesh from a geometrically associated neighborhood of points in a given fine mesh and the corresponding section of a given subdivision matrix **P**. This approximation is re-cast as the solution of a small underdetermined system of equations for each distinct format of row in a left inverse **A** for **P**. **A** and **P** are then used to generate **B** and **Q** to complete the analysis and reconstruction components of a biorthogonal system: **AP = I, AQ = 0, BP = 0, BQ = I**. These components comes from solving underdetermined equations, which define each row format in **B** and column format in **Q**, and the resulting filters are finite, with width depending on the original choice of neighborhood for determining **A**. We shall show how the construction can be cast entirely in terms of masks. A sample result is shown for cubic B-spline curve subdivision in the figure below. We have used the construction for subdivisions on curves, tensor products, triangular meshes, and irregular meshes. The techniques work equally well for the boundary nodes of a mesh and can also be employed for progressive mesh transformations. The biorthogonal systems are produced at the cost of being able to dictate the inner product on the underlying function spaces. Instead, an inner product is induced that can be extracted from the systems. The construction may yield unstable as well as stable multiresolutions, and we shall offer a quick check that indicates which is produced. Examples will be shown of our multiresolutions applied to several sets of points comprising images, free-form curves, laser ranged surfaces, and geographic data.

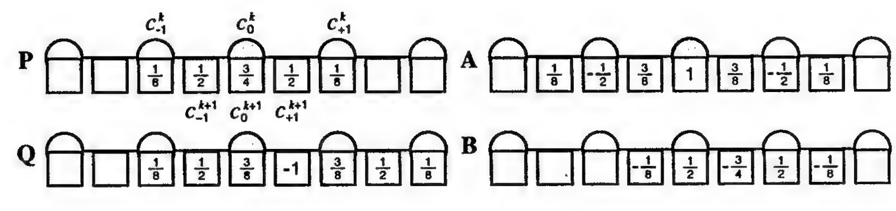

Fig. 1. A Stable, Cubic B-Spline Multiresolution.

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A General Scheme for Constrained Curve Interpolation

P. Costantini, M. L. Sampoli*

Keywords: *Spline interpolation, shape-preserving curves, abstract schemes, fairness functional.*

In the definition and development of mathematical methods for curve fitting and reverse engineering a great deal of research has been done in the field of *constrained* interpolation and approximation. Many applications indeed require the construction of a curve which satisfies, along with classical interpolation conditions, also other constraints given by the context where the curve is looked for. In industrial applications, for instance, feature lines of the roof of a car should not have undesirable bumps or wiggles but should be convex or network curves constituting the surface of the tail of an aircraft should not have any oscillations which could affect the aerodynamic properties of the resulting surface. In these cases the additional constraints are given by *smoothing* constraints or *shape-preserving* constraints. A way to deal with this problem is to construct an *ad hoc* method for every specific problem considered.

On the other hand we may notice that most schemes developed to solve constrained interpolation (as well as approximation) problems follow a common procedure which can be exploited to build up a unified approach suitable and applicable to various different problems. This approach is indeed given by abstract schemes, where the term *abstract* does not mean a generalization to abstract spaces but, on the contrary, actually refers to a general purpose practical theory that can give a common framework in which various and different problems and targets can be dealt with. It can be proven indeed that *any* problem regarding piecewise defined interpolating (or approximating) functions subject to *any kind of local constraints* can be modelled by means of abstract schemes and therefore solved with a general algorithmic procedure. Even if this algorithm can be seen as a general frame adaptable to a wide class of constrained problem, its practical application has been so far mainly confined to functional interpolation, because their practical applications (in their standard form) in the parametric setting are sometimes very cumbersome.

We have proposed a way to overcome these problems. In particular this talk describes the application of abstract schemes to construct shape-preserving interpolating curves. In contrast to the existing techniques, the present method uses piecewise defined curves whose pieces are taken from *standard* polynomial spaces without tension properties. Moreover with this approach many *optimization functional* (for instance any fairness functional) can be used to select the best interpolant among the admissible ones.

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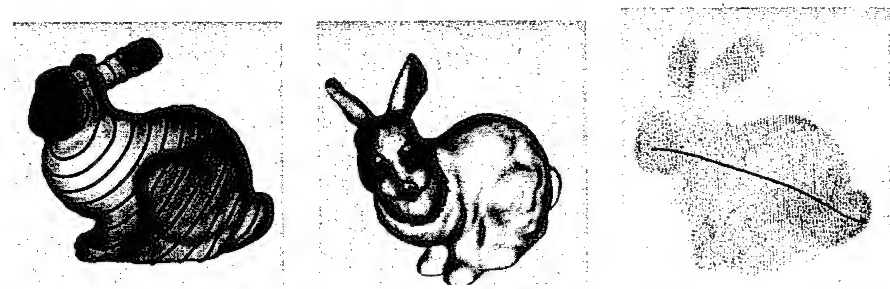
Distance Functions and Geodesics on Implicit and Unorganized Points Hypersurfaces

Facundo Mémoli, Guillermo Sapiro*

Keywords: *Implicit hypersurfaces, unorganized points, noise, distance functions, geodesics, Hamilton-Jacobi equations, fast computations.*

An algorithm for the computationally optimal construction of intrinsic weighted distance functions on implicit hypersurfaces is introduced in this paper. The basic idea is to approximate the intrinsic weighted distance by the Euclidean weighted distance computed in a band surrounding the implicit hypersurface in the embedding space, thereby performing all the computations in a Cartesian grid with classical and efficient numerics. Based on work on geodesics on Riemannian manifolds with boundaries, we bound the error between the two distance functions. We show that this error is at least of the same order as the theoretical numerical error in computationally optimal, Hamilton-Jacobi based, algorithms for computing distance functions in Cartesian grids. Therefore, we can use these algorithms, modified to deal with spaces with boundaries, and obtain also for the case of intrinsic distance functions on implicit hypersurfaces a computationally efficient technique. The approach can be extended to solve a more general class of Hamilton-Jacobi equations defined on the implicit surface, following the same idea of approximating their solutions by the solutions in the embedding Euclidean space. The framework here introduced thereby allows for the computations to be performed on a Cartesian grid with computationally optimal algorithms, in spite of the fact that the distance and Hamilton-Jacobi equations are intrinsic to the implicit hypersurface. The technique is also applicable to other surface representations like noisy unorganized points.

The figure shows the color mapped distance function from the tip of the nose on an implicit bunny (together with iso distance lines), the use of this technique for texture mapping on an implicit surface, and the distance function and geodesic on a surface defined by a set of unorganized points, without the need at all for intermediate surface reconstruction.



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Automatic Surface Modification Based on Finite-Element Node Displacements

Ramon F. Sarraga

Keywords: *Variational methods, morphing, spline interpolation and approximation.*

This talk presents a method for modifying trimmed B-spline surfaces automatically in accordance with displacements prescribed at a finite set of points in R^3, such as node displacements predicted by finite-element analysis. The method is based on the "morphing" approach introduced by Sederberg and Parry in 1986. The input to the method consists of (a) a set of trimmed B-spline surfaces and (b) a set of points and associated displacement vectors in R^3. A rectangular volume, enclosing the spline surfaces and the input points in R^3, is represented as a volume spline, i.e., a trivariate tensor-product spline. A modified volume spline is computed using a least-squares fit of the given point displacements together with a smoothing functional. The modified spline surfaces are defined as compositions of the original parametric functions and the modified volume spline (i.e., a "morphing"). To ensure compatibility with standard commercial CAD/CAM systems, the modified surfaces are fitted with appropriate splines using a standard fitting procedure applied in the parameter domains of the original surfaces.

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Approximation Order of Refinable Functions via Quotient Ideals of Laurent Polynomials

T. Sauer

Keywords: *Refinable functions, approximation order, quotient ideals.*

If the translates of a univariate refinable function admit the representation of polynomials of a certain degree, then this function must have a zero of a certain order at 1, i.e., a factor of the form $(z+1)^k$ for some $k > 0$. The multivariate counterpart of this property, even for arbitrary expanding scaling matrices is containment in a quotient ideal of a very peculiar form. Besides presenting this result, the talk shows how this fact can be used to construct minimally supported masks which satisfy a sum rule of a given order.
This is joint work with H. M. Möller.

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Interpolation Problems Using Conic Splines With Monotone Curvature.

L. Schiavon

Keywords: *Conic spline, quadratic (BR) curve, monotone curvature, G^k-continuity.*

Splines are widely used by CAD systems for the design process. A frequently encountered problem is the interpolation of planar data sets (points or tangent vectors). The fairness of the spline we aim to obtain may depend on two criteria:

(a) the monotonicity of curvature which provides to the spline an aesthetic shape [2],

(b) the smoothness of the spline (G^2-continuity) which preserves the curvature radius continuity [4].

Here, conic splines are used. The number of conic segments connected together N may vary resulting in a different number of data. Conics are quadratic rational curves and can be written in the Bernstein Rational (BR) form controlled by a set of weighted points [1]. In [3], Frey and Field characterized the monotonicity of curvature for quadratic (BR). From this result, the following interpolation problem is studied: find a conic spline ($N = 2$) which verifies criteria (a) and (b) where

(c) the boundary points and tangent vectors,

(d) an intermediate point or tangent vector,

are given.

From the data, we are able to say if there exist solutions (eventually one) or not. In the first case, we can characterize all the solutions and discuss about their geometric fairness (strict monotone curvature, minimal arc length or G^3-continuity). Moreover, we describe an algorithm for generating them. As the weights are used for the definition of each quadratic (BR), different parametrizations for a conic spline may be thinkable. We can choose one in order to obtain a C^2-solution.

Finally, we focus on the interpolation problems where two or three intermediate tangent vectors are given ($N = 3$ or 4). We are able to say if there exist solutions or not and characterize a particular one.

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Subdivision for Modeling and Simulation

Peter Schröder

Keywords: *Subdivision, digital geometry processing, algorithms, numerical simulation, compression, surfaces.*

Subdivision as a basic primitive for the construction of surfaces is now well established as a fundamental technique in computer graphics applications. Both the theoretical progress in the mathematical community and the algorithmic progress in the computational community have been enablers in this development.

In this presentation I will discuss some of these recent developments and illustrate what they have done for applications in Digital Geometry Processing. I will cover the construction of subdivision schemes based on repeated averaging, their use in geometry compression and in the construction of adaptive numerical solvers for PDEs.

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Two-scale Regularity and Sparse Grids for Homogenization Problems

Christoph Schwab

Keywords: *Two-scale FEM, homogenization, sparse two-scale FEM.*

We present a two-scale regularity theory for elliptic homogenization problems in d-dimensional domains D (d=1,2,3) with coefficients oscillating at period $e << \mathrm{diam}(D)$.

We show that the solutions u(x,e) allow for high regularity independent of e if they are viewed as traces of certain functions in 2d-dimensions.

We design a FEM on sparse grids in 2d-dimensions and establish robust in e convergence rates for it.

The complexity of the FEM is shown to equal, up to log(N), that of standard FEM in the physical d-dimensional domain on meshes of width $e < h < 1$.

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Spatial C^2 PH Quintic Spline Curves

Rida T. Farouki, Carla Manni, Alessandra Sestini*

Keywords: *Pythagorean–hodograph curves, quaternions, splines.*

The distinguishing property of a polynomial Pythagorean–hodograph (PH) curve $\mathbf{r}(t) = (x(t), y(t), z(t))$ is that it satisfies the Pythagorean condition $x'^2(t) + y'^2(t) + z'^2(t) = \sigma^2(t)$ for some polynomial $\sigma(t)$. Thus, PH curves offer significant advantages over "ordinary" polynomial curves in computer–aided design and manufacturing applications, including: the ability to compute arc lengths precisely (without numerical quadrature); rational offsets that are compatible with exact representation in CAD systems; exact real–time CNC interpolators for fixed or variable feedrates along curved tool paths; interpolants to discrete data with improved shape characteristics (more even curvature profiles). The lowest–order PH curves that can interpolate arbitrary first–order Hermite data are quintics [3,4]. The Hermite interpolation problem for *planar* data, and its extension to C^2 PH splines, have been studied using the complex variable model, which greatly facilitates the construction and shape analysis of interpolants [1,3,5].

Choi et al. [2] give an elegant characterization of *spatial* PH curves in terms of quaternions. In contrast with the planar case, it transpires that a single segment of a spatial PH quintic Hermite interpolant exhibits two degrees of freedom, which may strongly influence the shape of the curve [4]. In this talk, we first briefly discuss an efficient algebraic criterion to select the free parameters in the first–order Hermite interpolation problem so as to produce fair curves. Second, we show that constructing spatial C^2 PH quintic interpolating splines incurs a system of quadratic equations in quaternion unknowns with three degrees of freedom in each subinterval. We propose a strategy to determine these free parameters. With this choice, a solution of the quadratic system can be efficiently approximated, to within machine precision, by few iterations of a Newton–Raphson–like method. The quality of the resulting PH spline depends strongly upon the chosen starting point. A suitable starting approximation, that produces fair curves, is proposed, and empirical results from an implementation of the method are presented.

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Spline Implicitization of Planar Curves

B. Jüttler, J. Schicho, M. Shalaby*

Keywords: *Implicitization, B-spline, approximation, knot removal.*

Implicit and parametric representations are the two main ways to define planar curves. Each representation has its own advantages. The availability of both often results in simpler computation. For example, the intersection of two curves can be obtained easily if both implicit and parametric representations are available.

From classical algebraic geometry, it is known that each rational parametric curve has an implicit representation, while the converse is not true. The process of converting the parametric equation into implicit form is called implicitization. A number of established methods for exact implicitization exists such as resultant-based method, Gröbner bases method, and moving curve and surface method. However, exact implicitization has not found widespread use in CAGD. This is in part due to the facts that exact implicitization process is relatively complicated, especially in the case of high polynomial degree, and often produces large data

volumes. For example, the number of the coefficients of degree n planar curve is $2(n+1)$ in the parametric case while it is $(n+1)(n+2)/2$ in the implicit case. Due to these reasons, approximate implicitization has been proposed [1].

From above, we can conclude that it is important to keep the implicit degree as low as possible. In this work, a new method for constructing a low degree spline implicitization of planar curves is proposed. Spline implicitization consists of a partition of the plane into polygonal pieces, and an implicit polynomial for each piece. On the boundaries, these polynomial pieces are joined together with C^m continuity for suitable choice of m.

The main structure of the proposed method is: First, to ensure the low degree condition, quadratic B-splines, defined on a sufficiently high number of knots sufficient to guarantee the desired accuracy, are used to approximate the given parametric curve. Then, a data reduction technique is used to reduce the number of quadratic B-spline segments. In our implementation, we use spline wavelets for data reduction. The resulting quadratic B-spline segments are implicitized. Finally, by multiplying with suitable polynomial factors, these implicitized quadratic segments are joined together with C^1 continuity along suitable transversal lines. The behavior of these lines is analyzed and some numerical examples are given. We are planning to generalize the proposed method to the surface case, this is the topic of ongoing work. Our method yields a simple and powerful tool for the spline implicitization of planar curves.

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A Priori and a Posteriori Measurement of Parameterization Error

A. Sheffer

Keywords: *Parameterization, faceted surfaces, meshes, curvature.*

Parameterization of 3-dimensional faceted surfaces is an active research problem in modeling and manipulation of mesh surfaces [1-4]. It has many applications in computer graphics, finite-element meshing, surface reconstruction, and a whole range of engineering applications. Most methods use the plane as the parameterization domain and concentrate on reducing the metric distortion occurring during the parameterization. Most works define their own measure of distortion, and minimize it. As a result each method tends to do best within its own metric. This raises the question of providing more general distortion metrics to enable objective comparison of the methods. This topic will be addressed in the first part of this talk. Several possible measures will be presented and applied to some of the popular parameterization method results.

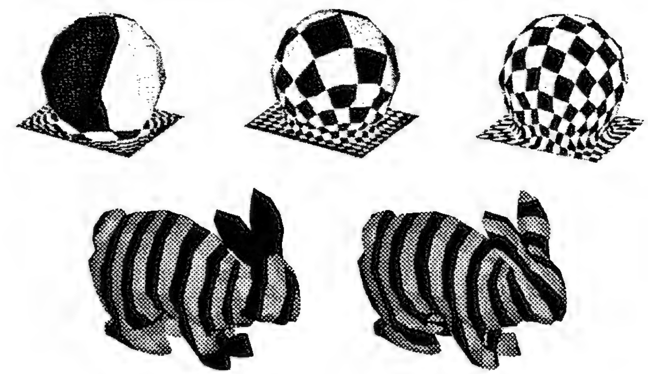

Fig. 1. Different parameterizations. Row one (left to right): harmonic [1], convex combinations [2], ABF [3]. Row two: MDS [5], seams cutting combined with ABF [4] .

Only developable surfaces, i.e. surfaces with zero Gaussian curvature can be parameterized with zero distortion. This raises the question of how to measure the parameterization error a priori, i.e. how to provide a lower bound on the distortion independent of the parameterization method used. This measure is crucial for applications that subdivide the surface into parameterizable patches. It is also important to estimate the performance of parameterization methods, as clearly the methods can not perform better than this bound. In the second part of the talk several a priori distortion measures will be presented. The advantages and disadvantages of them will be desribed. The measures will be compared with the achieved a posteriori distortion using different methods.

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Constructing B-spline Surfaces from Multiple Images

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Keywords: *B-spline surface fitting, projective vision, parameterization.*

Recent developments in projective vision theory allow 3D structures to be reconstructed from multiple images of a scene. In many applications, it is often desirable to use a more compact surface representation than the raw points or the triangular mesh of the points. B-spline surface is a good choice. In this talk, we present a method for constructing B-spline surfaces from multiple images. One of the major difficulties in fitting B-spline surfaces to dense point sets is the parameterization problem. Since an image is a projection of the 3D points on a surface to a plane, the image serves as a parameterization. Multiple images can be combined into a single image by using mosaic techniques. In this way, the combined image becomes a natural parameterization.

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Extreme Simplification using Multiple Billboards

Xavier Decoret, François Sillion*

Keywords: *Extreme simplification, mesh decimation, image-based impostors.*

We introduce "billboard clouds" – a new approach for extreme simplification. Models are simplified onto a set of planes with texture and transparency maps. We present an optimization approach to build a billboard cloud given a geometric error threshold. After computing an appropriate density function in plane space, a greedy approach is used to select suitable representative planes. A very good surface approximation is ensured by favoring planes that are "nearly tangent" to the model. This method does not require connectivity information, but still avoids cracks by projecting primitives onto multiple planes when needed. The technique is quite flexible through the appropriate choice of error metrics, which can include image-space or object-space deformation, as well as any application-dependent objective function. It is fully automatic and controlled by two intuitive user-supplied parameters. For extreme simplification, our approach combines the strengths of mesh decimation and image-based impostors. We demonstrate our technique on a large class of models, including smooth manifolds and composite objects, as well as entire scenes containing buildings and vegetation.

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Tangent Plane Continuity between Adjacent Parametric Surfaces

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Keywords: *G^1 continuity, B-spline surfaces, conditions for construction.*

A CAD model normally consists of several surfaces, some of which having a smooth transition. The problem of attaching one or more surfaces with exact or approximate G^1 continuity to a set of already existing primary surfaces, is addressed. This problem appears for instance when filling a hole between surfaces. Necessary conditions on the topology of the existing surfaces in order to achieve exact G^1 continuity between the new and the existing surfaces, are presented. Boundary conditions for the new surface(s) are computed. Situations where the topology of the primary surfaces denies G^1 continuity or where too strict continuity requirements lead to an unfortunate shape of the new surface, are discussed. In some situations approximate tangent plane continuity yields a better overall model than exact continuity would provide. Higher order continuity than G^1 can be of interest if the primary surfaces have internal continuity C^2 or higher. Often this is not the case, and even when conditions on G^n continuity can be deduced, shape problems due to strict continuity requirements are more apparent in this case. The talk concentrates on polynomial B-spline surfaces, but most of the considerations apply to any parametric surfaces.

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Automatic Contour Line Recognition From Scanned Topographic Maps

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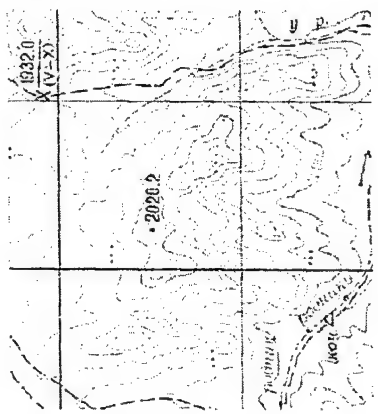
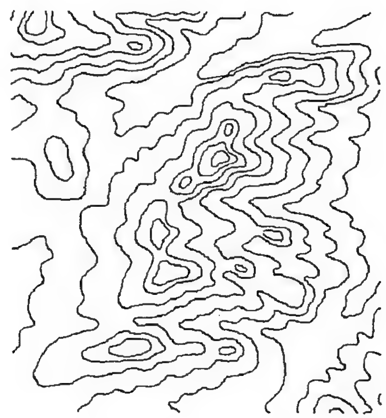
Keywords: *Image processing, pattern recognition, seeded growing regions.*

In this paper, we present a new method for the automatic recognition of contour lines starting from scanned topographical maps. This is a difficult problem due to the presence of complex textured backgrounds and information layers overlaid on the elevation lines (e.g. grid lines, rivers, roads, buildings, etc.). Our approach is essentially based on the global topology of the contour lines (i.e. a set of non-intersecting closed lines). The method starts from a map scanned in RGB. The image is first segmented by a color classification process. The object is than thinned and vectorized with an original method based on growing regions and weighted graph.





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Multi-Resolution Splines for Rendering

Michael M. Stark*, Elaine Cohen, Tom Lyche, Richard F. Riesenfeld

Keywords: *Splines, rendering, multiresolution methods.*

Tensor product B-spline functions have an established place as an effective approximation method for multi-variate functions. Surface irradiances and radiances in synthetic images are multivariate functions amenable to tensor product approximation, but difficulties occur when there is high-frequency detail such as along shadow boundaries or in highly specular reflections. Tensor product functions are not suited to the type of local refinement that would be necessary for a compact representation in such regions. In this work, we examine the use of multi-resolution splines for efficiently representing rendered surface functions.

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Coorbit Spaces and Banach Frames on Homogeneous Spaces

S. Dahlke, G. Steidl*, G. Teschke

Keywords: *Frames, nonlinear approximation, modulation spaces, spheres, square integrable group representations, time - frequency analysis.*

A classical problem in applied mathematics is to analyze and to process a given set of signals. In this context, the short–time Fourier transform and the wavelet transform have been established as powerful tools in signal analysis. Although these transforms may behave quite different in applications, there exists a common thread between Gabor and wavelet transform. Both can be derived from square integrable group representations of a certain group. In any case, when it comes to practical applications, only a discrete set of coefficients can be handled. It is therefore necessary to dicretize both transforms to obtain some kind of basis for the function space under consideration. However, constructing

some stable basis may be asking to much, nevertheless, it is usually possible to obtain at least a frame.

This paper is concerned with the construction of generalized Gabor frames on homogeneous spaces. The major tool is a unitary group representation which is square integrable modulo a certain subgroup. By means of this representation, generalized coorbit spaces can be defined. Moreover, we can construct a specific reproducing kernel which, after a judicious discretization, gives rise to Banach frames for these coorbit spaces. We also discuss nonlinear approximation schemes based on our new Banach frames. As a classical example, we apply our construction to the problem of analyzing and approximating functions on the spheres.

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Multiresolution Representation and Subdivision on Curves and Surfaces in Symmetric Spaces

David Donoho, Nira Dyn, Peter Schröder, Victoria Stodden*

Keywords: *Multiresolution, subdivision, Lie-valued data, symmetric spaces.*

Innovative measurement schemes now provide data on equally-spaced grids in time or space where the data values have rich algebraic and geometric content. Examples include: orientations of aircraft or of human body parts as a function of time, covariance matrices as a function of space or time, direction fields as a function of space, or time; families of planes indexed by space.

In each case, the new types of data values belong to a specific manifold. For example, orientations belong to the Lie Group SO(3); covariance matrices belong to the Jordan Algebra of positive definite matrices; and families of planes belong to the class of Homogeneous spaces called Grassman manifolds. These manifolds are all naturally regarded as instances of symmetric spaces, that is, Riemannian manifolds with transport-invariant curvature tensors.

We describe here a class of multiscale methods which provide for data taking values in symmetric spaces what wavelets provide for ordinary data taking values in the reals. These methods extend classical multiscale compression and noise removal, to such cases of structured data.

At the heart of our methods are two-scale refinement schemes that interpolate data to midpoints of existing grid points by transforming values at several neighboring grid points into the tangent plane of the symmetric space, performing a classical refinement scheme in that vector space, and using the exponential map to transfer the results back to the manifold. The resulting schemes respect the algebraic and geometric properties of the data manifold; for example, geodesics have all their 'wavelet coefficients' vanishing, and hence are highly compressible.

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On the Construction of Tight Affine Frames on Bounded Intervals

Joachim Stöckler

Keywords: *Affine frame, dual spline, approximate dual.*

Affine frames on bounded intervals are constructed from a spline multiresolution analysis, which is defined by a nested sequence of spline spaces with non-uniform knot sequences and multiple knots at both endpoints. The key step is the definition of "approximate" B-spline duals which have small support and reproduce all polynomials in the spline space. These approximate duals take on the role of the vanishing moment recovery function, that was introduced in [1] for the construction of affine frames on the real line. A new representation of the dual basis of Bernstein polynomials $B_{n,k}$ of degree n is obtained as a special case of these duals. In particular, we show that for all $1 \le k, \ell \le n$, we have

$$(n+1)\sum_{i=0}^{n}\frac{(n-i)!}{i!n!}\int_0^1 x^i(1-x)^i\frac{d^i}{dx^i}B_{n,k}(x)\frac{d^i}{dx^i}B_{n,\ell}(x)\,dx=\delta_{k,\ell}.$$

Our construction encompasses all types of knot refinements, where a different number of "new" knots may be inserted per interval, and simple or multiple

knots can be mixed. In this regard, our construction provides a general approach that encompasses non-uniform refinements as well as the "multiwavelet" setting, where several generators of the multiresolution analysis are given as B-splines with multiple knots. Examples for tight spline frames of degree up to 4 with simple and multiple knots are presented.

This is joint work with C. K. Chui and W. He.

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Accuracy and Algorithmic Issues in Surface Parameterization

Eric de Sturler

Keywords: *Surface parameterization, angle based flattening, metrics, optimization.*

Surface parameterization is currently a topic of significant interest with applications in many areas, for example meshing and graphics. The main reason is that working with a flat representation of a 3D surface mesh is much easier than working with the 3D mesh itself and it allows much stronger quality criteria to be satisfied. Since we work on the 3D surface implicitly it is important that we take the appropriate transformations that result from the mapping from the 2D parameter domain to the 3D surface (or vice versa) into account.

Since no mapping exists from a non-smooth 3D surface to the plane that has no linear, angular, and areal distortion we need to take the appropriate transformations or sufficiently accurate approximations to these into account to generate meshes or other structures (such as textures) on the 3D surface with the required accuracy. As long as the Jacobian of the mapping is well-defined exact metrics on the 3D surface can be computed; however, these may come at a significant cost. For example the (shortest) distance between two points on the 3D surface is not readily computed because the shortest path is not given by any obvious curve or line. We need to minimize the integral over a piece-wise constant (matrix) function on the 2D parameter domain to find that path and its length. In order to derive algorithms with acceptable cost (this varies by application) we need to make approximations to the Jacobian of the mapping, a piece-wise constant matrix function. The properties that we want from such an approximation again depend on the application.

We will discuss the effects of such approximations on the metrics and their error on the 3D surface. We will also discuss the inherent consequences of certain (desirable) properties of an approximation (e.g., continuity or differentiability). Finally several methods have been proposed to compute parameterizations. These methods are based on different principles and they create different types of distortions. We will discuss the consequences of those distortions; to compute and use accurate metrics at acceptable cost on the 3D surface is easier with some distortions than with others, and this may guide our choices in the computation of the parameterization.

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Curvature Measures for Discrete Surfaces

John M. Sullivan

Keywords: *Discrete surfaces, optimization.*

There is a well-known interpretation of Gaussian curvature for discrete (triangulated) surfaces [1]. It is natural because it preserves the Gauss–Bonnet theorem, which equates the integral of Gaussian curvature to a boundary integral. Less familiar are analogous boundary integral relations for mean curvature, including the equation

$$\int_{\partial D} \eta \, ds = \int_{\partial D} \nu \times d\mathbf{x} = 2 \iint_D H\nu \, dA = 2 \iint_D \mathbf{H} \, dA,$$

which can be understood as a balance of physical forces.

We will show how to use such relations to guide the proper interpretation of mean curvature (and other geometric quantities) for discrete surfaces. This new understanding helps to explain the theory of discrete minimal surfaces [5]. It also elucidates why, in early work [3,4] on simulations of Willmore energy ($W = \int H^2 \, dA$), certain discretizations were better than others. Minimizing W can be useful for fair surface design [6].

Similarly, for space curves, some quantities, like total curvature or even writhe [2], have natural interpretations for polygons. We will examine further cases, like knot energies and ropelength [7], where a proper consideration of the geometry involved can lead to a natural discretization.

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On Cubic Algebraic Curve Interpolation with Geometric Contraints

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Keywords: *Cubic algebraic curve, geometric contraints, interpolation.*

Given four points $p_i(x_i, y_i)$, $(i = 1,2,3,4)$ which can construct a simple quadrilateral with counterclockwise direction and given two lines L_1 and L_2 passing the endpoints p_1 and p_4 respectively.

Denote a line passing points p_2 and p_3 by L_3, a line passing points p_1 and p_2 by L_4, a line passing p_3 and p_4 by L_5, a line passing points p_4 and p_1 by L_6.

We call the area constructed by lines L_1, L_2 and L_6 **control area**. Generally, there are two control areas constructed by three lines L_1, L_2, L_6 on the two sides of L_6.

Let $p(x,y)$ be any point on the plane. Then the equations of lines L_i $(i = 1,2,3,4,5,6)$ have the following forms

$$l_i := l_i(p) := l_i(x,y) = a_i x + b_i y + c_i \quad (i = 1,2,3,4,5,6),$$

which satisfy the following conditions:

(i) $a_i^2 + b_i^2 = 1 \quad (i = 1,2,3,4,5,6)$,
(ii) $l_1(p_4) > 0, \quad l_2(p_1) > 0$,
(iii) for any inner point p' of the quadrilateral, $l_i(p') > 0$ $(i = 3,4,5,6)$.

Let λ be a real number. Define the function:

$$f_\lambda(p) = f_\lambda(x,y) = (1-\lambda) l_1(x,y) l_2(x,y) l_3(x,y) + \lambda l_4(x,y) l_5(x,y) l_6(x,y) \quad (\lambda \neq 0,1).$$

Now define a cubic algebraic curve

$$C(\lambda) : f_\lambda(x,y) = 0. \tag{1.1}$$

We can verify easily that the four points p_1, p_2, p_3, p_4 are on the curve $C(\lambda)$ and that L_1, L_2 are the tangent lines of $C(\lambda)$ at p_1 and p_2 respectively.

There are many researches on cubic algebraic curves. Here we only cite some of them [1-2]. Recently Zhang et el.[3] researched the cubic algebraic curve (1.1) and established some results.

In this paper we further investigate the cubic algebraic curve (1.1) and give the necessary and sufficient conditions on which the curve $C(\lambda)$ has a continuous and convex branch passing points p_i $(i = 1,2,3,4)$.

The following is our main result.

Theorem Suppose that four points p_1, p_2, p_3, p_4 costruct a simple quadrilateral with counterclockwise direction. Given two lines L_1, L_2 passing the endpoints p_1, p_4. Then for $0 < \lambda < 1$ there exists a continuous convex branch of the cubic curve $C(\lambda)$ passing $p_i (i = 1,2,3,4)$ and with tangent lines L_1, L_2 at p_1, p_2 respectively if and only if the quadrilateral $p_1p_2p_3p_4$ is convex and it is completely on a control area constructed by lines L_1, L_2, L_6.

We also investigate some other cases when the quadrilateral is not convex and it is not completely on a control area.

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Spectral Method of Curves Nodes Distribution with B-splines Interpolation

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Keywords: *Curve, polynom, spline, node, spectrum.*

The new method of arrangement of curves nodes, which given in the form of interpolation polynomial splines, is offered. In its basis lays the criterion which links integral power expressions in a curve space and in the spectrum space of B-spline sequences, interpolating these curves. These relations allow to define intervals between nodes even at nonequidistant case, and also in dependence of spline degree and values of curvature.

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A Wavelet-based Approach to Harmonic Transformation

Yuan Y. Tang*, X. C. Feng

Keywords: *Harmonic transformation, wavelet collocation.*

Geometric transformation plays a significant role in pattern recognition, image processing and related disciplines. Harmonic transformation is a very important geometric transformation. However, its implementation is very complicated, because the harmonic transformation can not be described by any fixed functions in mathematics. In fact, it is represented by partial differential equation with a given boundary condition. To solve such a equation efficiently, some typical numerical methods, such as finite element method and finite difference method, are discussed, and their shortcomings are analyzed as well. Based on the analysis, in this paper, a novel approach based on the integral equation and wavelets, called integral equation - wavelet collocation (IEWC) method, is presented. The IEWC consists of three phases. In phase 1, the partial differential equation is converted into boundary integral equation and representation by an indirect method. In phase 2, the boundary integral equation and representation are changed to plane integral equation and representation by boundary measure formula. In phase 3, the plane integral equation and representation are then solved by wavelet collocation method. Our approach has two main advantages, the pixels to be transformed is arbitrary and the program code is independent of the boundary. The performances are evaluated by experiments. Some examples are shown in Figures.

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The Thresholding Greedy Algorithm, Greedy Bases, and Duality

S.J. Dilworth, N.J. Kalton, D. Kutzarova, V.N. Temlyakov*

Keywords: *Greedy basis, quasi-greedy basis, democratic basis, concervative basis, bidemocratic basis, duality.*

Some new conditions that arise naturally in the study of the Thresholding Greedy Algorithm are introduced for bases of Banach spaces. We relate these conditions to best n-term approximation and we study their duality theory. In particular, we obtain a complete duality theory for greedy bases.

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A New Method for Computing a Composite PS Finite Element of Class C^k

A. Mazroui, D. Sbibih, A. Tijini*

Keywords: *PS finite elements, Hermite interpolants, decomposition.*

Let Δ be the equilateral triangulation of the plane $\mathbb{R}^2$. Let Δ_6 be the PS subtriangulation of Δ obtained by connecting each vertex to the midpoint of the opposite side in each triangle T of Δ. If $T = A_1A_2A_3$ is a triangle of Δ, then we denote $P_n^k(T, \Delta_6) = \{s \in C^k(T) \ : \ s_{|t} \in P_n, \ \forall t \in \Delta_6 \cap T\}$, where P_n is the space of bivariate polynomials of total degree $\leq n$.

For a smooth function u, it is well-known (see e.g., [1], [2] and [3]) that there exists a unique composite finite element u_k of class C^k, of degree $n(k) = 2k$ (resp. $2k+1$) for k odd (resp. even), which satisfies the following interpolation conditions :

$$D^\alpha u_k(A_i) = D^\alpha u(A_i) \quad \text{for } \alpha = (\alpha_1, \alpha_2) \in \mathbb{N}^2,$$
$$|\alpha| = \alpha_1 + \alpha_2 \leq k + [k/2] \text{ and } 1 \leq i \leq 3.$$

We propose in this talk a new method for computing recursively the composite finite element u_k. We show in particular that u_k can be written in the form :

$$u_k = u_0 + v_1 + \cdots + v_k,$$

where v_j, $1 \leq j \leq k$, are detail functions (or detail finite elements) of class C^{j-1} and degree $n(j)$ on T, and u_0 is the affine finite element of class C^0 which interpolates the values of u at vertices of T.

The above decomposition of u_k requires the expressions of the fundamental Hermite basis functions associated to each finite element $u_j = u_0 + v_1 + \cdots + v_j$, $1 \leq j \leq k$. Since the determination of these functions is complicated, we give here their explicit formulae and therefore numerical tests only for $k \leq 2$.

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Projective and Quaternionic Reconstruction of Objects

Leonardo Traversoni

Keywords: *Quaternions, splines, projective geometry, animation.*

It is possible to have a quaternionic version of the homogeneous representation of elements like points, lines and planes as well as represent the operations of projection and section used by projective geometry via geometric algebra using quaternions. Given that, what we do is to reconstruct a 3D surface using images taken with still or moving cameras. If we add to that the approximation via quaternionic splines of the movement of the surface we are able to represent and animate virtual movements of a 3D object with relatively few data given in a way that using other techniques will take a long time and effort to put in a single format to perform the calculations.

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Overview of Powell–Sabin Spline Subdivision and Wavelets

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Keywords: *Wavelets, lifting, Powell–Sabin splines, B–splines, subdivision.*

We give an overview of the current status of the research on Powell–Sabin spline subdivision and Powell–Sabin wavelets.

On triangulations with equilateral triangles a subdivision scheme exists: we can find a representation of the same spline on a uniform refinement of the triangulation. This subdivision scheme enables us to place the PS-surfaces in a multiresolution context. We use the subdivision scheme as the prediction step in the lifting scheme and add an update step to construct corresponding wavelets. We now derive a similar subdivision scheme for uniform triangulations with all triangles equal but not necessarily equilateral. Again the lifting scheme can be used to develop a wavelet transform on these triangulations. Currently new subdivision schemes are developed and also the irregular case is considered.

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Constrained Fitting for Multiple Surfaces

Tamás Várady*, Pál Benkő

Keywords: *Multiple surface fitting, reverse engineering, constraints.*

Reverse engineering is the process of converting 3D, multiple view, measured data into an evaluated boundary representation model, which can directly be transferred to CAD/CAM systems. In this talk we deal with conventional engineering objects bounded by simple analytic surfaces, such as planes, cylinders, cones, spheres, tori and blending surfaces. The bounding surfaces must be accurate, their surface type and the related parameters must be reliably estimated including complex linear extrusion surfaces and surfaces of revolution as well.

Automatic segmentation for this class of objects is possible [1,2], but individual surface fitting to the segmented point regions generally leads to (i) algorithmic difficulties at model building and (ii) insufficient model quality from engineering point of view. We are dealing with noisy measured data and numerical procedures, thus the surfaces will be inaccurate. For example, often "almost" tangential surfaces will be obtained, which will cause problems at surface-surface intersections, when smooth edges need to be determined. Another difficulty is the occurrence of little artificial edges at vertices with more than three edges.

General "beautification" may be necessary to enforce a wider range of engineering constraints, such as parallelism, perpendicularity, tangency, concentricity, or symmetry amongst various geometric entities as well.

The constrained fitting technique is based on a set of surfaces, and segmented data points to be approximated. Each entity is characterized by a hypothetical surface type and unknown parameters. There is a given set of constraints (specified by the user or derived by the reverse engineering system itself), what we want to satisfy, i.e. we would like approximate simultaneously several surface elements, while the constraints are also satisfied. A new method is presented, which is computationally efficient and capable of resolving conflicts between constraints. Computational considerations of applying so-called auxiliary elements, and speeding up the computation by separating the terms holding the point data and the surface parameters will also be discussed.

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Evolution of Apparent Contours of Smooth Surfaces

Gert Vegter

Keywords: *Apparent contours, parallel projection, Morse theory, singularity theory, local models.*

The contour generator of a smooth surface in three-space, under parallel or perspective projection, locally separates backward and forward facing regions on the surface. It consists of those points on the surface where the normal is perpendicular to the direction of projection. The apparent contour is the projection of the contour generator onto the image plane.

The apparent contour and the contour generator are important visibility features of a smooth surface, e.g., in connection with visualization and non-photorealistic rendering. In computer vision, techniques have been developed for partial reconstruction of a surface from a sequence of apparent contours corresponding to a discrete set of nearby projection directions.

Generically, the contour generator is a smooth curve consisting of several components. These components may merge or split when the projection direction changes. In this talk we review an algorithm for the computation of the contour generator of implicit surfaces, and describe generic evolutions of the apparent contour under varying directions of projection.

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Multiresolution Mesh Generation using Combined Simplification/Refinement

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Keywords: *4-8 meshes, adaptation, hierarchical data structures.*

We present a method for constructing a multiresolution mesh with nice properties for two-dimensional manifold surfaces in the Euclidean tridimensional space.

The input surface can be specified in various forms, such as parametric, implicit or even volumetric descriptions.

The method employs a combination of simplification and refinement to obtain a multiresolution mesh structure with the following desirable properties: (1) bounded aspect ratio of triangles in the mesh; (2) underlying semi-regular 4-8 hierarchical structure; (3) geometric approximation within a prescribed tolerance.

The strategy adopted by the method consists of using simplification to obtain a base mesh with good aspect ratio, followed by adaptive 4-8 refinement to produce a multiresolution mesh that approximates the surface and possesses an underlying regular structure.

The base mesh construction uses topological simplification of discrete neighborhoods. The input surface is processed in order to compute a volumetric distance function. Then, a polygonization algorithm of the isosurface produces a connectivity graph. A restricted disk covering is generated along with the corresponding Voronoi diagram. The base mesh is the Delaunay triangulation of this disk covering.

The multiresolution structure is produced by applying adaptive 4-8 refinement to the base mesh. At each step, the current mesh is adjusted by three types of warping forces that respectively improve: geometry;' parametrization and smoothness.

Distance functions are used throughout the method. Intrinsic distance over the connectivity graph is used to compute the disk covering. Extrinsic distance over the ambient space is used to compute the adaptive approximation.

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Three-Dimensional Digital Surface Reconstruction

Y.A. Vershinin

Keywords: *Curves and surfaces in industry, computer vision, image processing, B-splines, visualization.*

Most anthropometric measurements today based on using such standard tools as anthropometers, calipers and tape measures. Automatisation of measurement process with high speed and accuracy using non-contact scanners can give significant benefit for industry and for customers. The problem of digital surface reconstruction and surface measurement is very important also in medicine: for facial surgery simulation, diagnosis of craniofacial anomalies, trauma to the head, for reconstructive surgery.

Different approaches and equipment have been developed for the solution of this problem, for example, the photogrammetric method based on the principle of rasterstereography [2], a moire - based light projection system [4], and the 3D reconstruction system [1]. All these methods are based on non-contact measurement principles. The surface digitization system based on a contact method is presented in [3]. The systems described above have many unresolved problems, (such, for example, as a time consuming analysis of the pattern of the object in order to obtain its digital image [2]; a lengthy calibration procedure and intensive computation (off-line processing of data) [4]; huge size and weight, high cost of the system [1]; slow data acquisition and restricted accuracy due to friction of a probe with a surface [3]).

Thus the need of a simple, cheap and fast 3D digital surface reconstruction system still exist today. The system presented in the paper allows to solve these problems. The system has been developed and built using different measurement systems, which were not originally constructed for the realization of the relevant problem. The laser scanning system works as follows: The laser projects a narrow line onto a surface of an object. The object is rotated on the rotary table. Two CCD cameras record position of the line according to the angle of rotation of the object. The frame grabber and the PC computer process under software control images from the cameras and record co-ordinates pf points where the light falls on the surface of the object. Data obtained are stored on the hard disk of the computer and then digital image is reconstructed by the program.

The idea of extracting co-ordinates of surface points by a laser line and the description of the calibration procedure are given in the paper together with the algorithm of 3D digital surface reconstruction. The comparison of images created using rectangular and triangular polygons is provided. The wire-frame of the reconstructed image of the human face has been rendered in order to represent the realistic color image. It is emphasized in the paper that the representation of an object by biparametric cubic surfaces (B-splines) gives big reduction of data points [5]. Smooth curves also give considerable advantages in visual representation of complicated surfaces.

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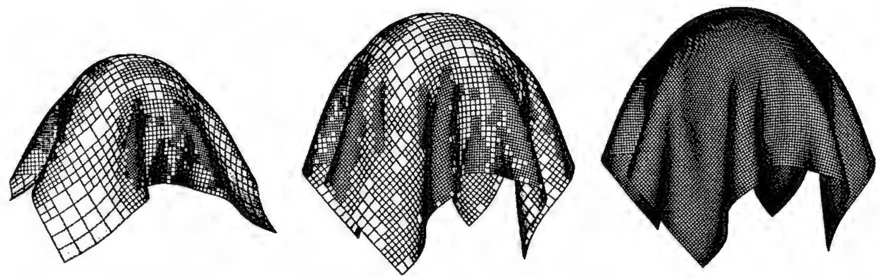
Cloth Simulation Using Adaptative Meshing

Julien Villard*, Houman Borouchaki

Keywords: *Adaptative meshing, draping, cloth simulation, particle system.*

Cloth simulation methods are based, in general, on mechanical models [3][4]. The cloth is geometrically defined by a uniform grid. Fabrics being a very flexible material, wrinkles appear on its surface when submitted to free or constrained motion. The main problem of the simulation is to represent realistically cloth surface motion. This is strongly dependent on the surface discretization. We present a new cloth animation scheme based on adaptative surface discretization. It can be seen as a multi-grid method which allows us to obtain realistic simulations. Our mechanical model is inspired by the Provot [3] model, a particle network linked together by springs. The mechanical properties of cloth : stretching, shearing and bending are modeled using springs. Stretching and shearing forces are computed using a modified Hook's law taking into account multi-grid mesh. A new computation of the bending forces, based on beams mechanical beheviour, is introduced. This model is well adapted to a multi-grid particles system. External forces are also applied to the system : gravity, air viscosity and so on. The mass of each particle is proportional to the area occupied by the associated node in the mesh. Euler's method is used to solve the ODE problem. The cloth surface is initially discretized with uniform quad elements oriented along the warp and weft directions. The adaptative method consists in subdividing the

four quads sharing a node if the surface is wrinkled in the neighbourhood of the node. Virtual nodes are then added, to ensure the conformity of the mesh with respect to the mechanical model (which take into account all the mesh nodes). The proposed scheme is stable with respect to the mesh element size and it preserves moments. The following example shows a piece of cloth falling onto a sphere. The result is quite similar to that obtained using a fine uniform mesh.



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Tangent and Curvature Continuous Matching of Surface Patches from the Practical Point of View

S. Wahl

Keywords: *Curves, surfaces, tangent continuity, curvature continuity.*

The continuity conditions between adjacent surfaces play an important part in the field of free form surface modelling in industrial design.

The focus is on tangent and curvature continuous transitions which need not necessarily be exact in the strict mathematical sense. More important are styling features, e.g. the harmonic course of planar sections or reflection lines across the common surface boundaries.

In this talk some principles from a practical point of view are presented. The examples are taken from Bézier and B-Spline curves and surfaces.

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CAGD Approximation and Interpolation in 2-Manifolds

Marshall Walker

Keywords: *2-manifold, geodesic, CAGD, de Casteljau, de Boor, Lagrange.*

Given a sequence of points which lie in a differentiable 2-manifold embedded in R, we propose a method for construction of approximating or interpolating curves that respect intrinsic geometry. The curves reproduce geodesic arcs. They are analogs of Bézier, B-spline, a nd Lagrange interpolatory curves, and are constructed by substituting geodesic arcs for line segments in the de Casteljau, de Boor and Atiken algorithms. An efficient algorithm for the calculation of geodesics is presented.

For a differentiable Riemannian manifold M and a point $x \in M$, it is well known that there is a neighborhood V of x in M such that for every $y \in V$, there exists a geodesic in M connecting x with y. This is the starting point. In the case that the points to be approximated or interpolated lie in an appropriate neighborhood, natural generalizations of the de Boor, de Casteljau, and Aitken algorithms are presented for the construction of analogs of Bézier, B-spline, or Lagrange interpolatory curves which lie in the manifold M. The algorithms are constructed from the corresponding Euclidean algorithms by substituting line segments with geodesic arcs. One would like to know that the standard repertoire of CAGD techniques for manipulating curves also have natural generalizations. Early results indicate that this is not the case. Many standard techniques depend explicitly on the affine and algebraic structures of Euclidean three space for which there is no apparent counterpart for manifolds. In particular there seems in general to be no succinct representation of manifold curves by basis functions. Thus,there are many aspects of the theory of manifold curves that remain open questions.

Although curves constructed lack the versatility of their Euclidean cousins, they do however possess the important characteristic that they respect aspects of intrinsic geometry. In particular, should control/interpolation points lie on a geodesic, then so also does the constructed curve. Constructed curves also satisfy a convex hull property in which the description of the Euclidean convex hull is slightly altered to provide a notion of geodesic convex hull. In order to effectively

construct the proposed curves, an algorithm for the calculation of geodesics is presented and applied to the construction of these curves on well-known 2-manifolds. In the case of a sphere, examples are presented which demonstrate the construction of curves which lie over the sphere.

The methods introduced have been touched on by Shoemake [3] in which he uses properties of quaternion arithmetic to describe curves on the quaternion sphere. Leversly and Ragozin [2] using different methods speak of Lagrange interpolation in a differentiable manifold.

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Robust and Efficient Computation of the Closest Point on a Spline

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Keywords: *Parametric cubic spline curve, distance, closest point, real-time simulation, road modeling.*

Parametric cubic spline curves provide a natural basis for modeling the geometry of road surfaces in real-time driving simulators. In many simulators, roads are represented by space curves that define a central axis or spine of a ribbon-like surface. The ribbon defines a curvilinear coordinate system in which points in space are expressed in coordinates of distance along the central axis, offset from the axis, and loft from the road surface. Simulators typically must map from global cartesian coordinates to local road coordinates at very high frequencies. A key component of this mapping is the computation of the closest point on the central axis from a point expressed in cartesian coordinates. To meet the stringent demands of real-time simulation, these computations must converge to an accurate solution with very high reliability in a small, fixed period of time. This paper investigates a two-step method to compute the closest point to a parametric cubic spline that exploits the complementary strengths of two optimization methods: Newton's method and quadratic optimization.

Our goal is to efficiently compute a point p_1 on a parametric cubic spline, $Q(s) = (x(s), y(s), z(s))$, that is closest to a point p_0. There is no closed-form solution to this problem. Therefore, we compute this distance through optimization. The distance between p_0 and any point p on $Q(s)$ is a function of the parameter s. Optimization is used to solve for the minimum distance between p_0 and p. The value of parameter s which optimizes this function gives the point p_1.

In this paper, we analyze the performance of Newton's method and quadratic optimization to find the closest point to a parametric cubic spline. We find both methods to be inadequate for real-time applications when used alone. Newton's method relies on having a good initial approximation to the solution. The quadratic optimization sometimes diverges or converges very slowly, even when the curve has a simple, smooth shape. These cases are sufficiently common to create catastrophic failures in simulations that must perform the computation many thousands of times in a single run. By comparing the convergence properties of the two methods, we discover that we can overcome their weaknesses by using the methods in combination. This observation leads us to a composite method in which the distance between the point p_0 and the curve $Q(s)$ is computed with a small number of iterations of quadratic optimization method followed by a small number of iterations of Newton's method. The quadratic method gives a coarse solution that is sufficiently accurate to provide a good initial approximation for the Newton's method. A small number of iterations of Newton's method refines the estimate. The speed of convergence is greatly improved and the likelihood of divergence nearly vanishes. In our experience with thousands of tests, the composite method always converged to an accurate estimate in a small number of iterations.

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A Subdivision Scheme for Surfaces of Revolution

G. Morin, J. Warren*, H. Weimer

Keywords: *Subdivision, surface of revolution, catmull-clark, repeated average.*

This talk describes a simple and efficient non-stationary subdivision scheme of order 4. This curve scheme unifies known subdivision rules for cubic Bsplines, splinesintension and a certain class of trigonometric splines capable of reproducing circles. The curves generated by this unified subdivision scheme are a splines whose segments are either polynomial, hyperbolic or trigonometric functions, depending on a single tension parameter. This curve scheme easily generalizes to a surface scheme over quadrilateral meshes. The authors hypothesize that this surface scheme produces limit surfaces that are C^2 continuous everywhere except

at extraordinary vertices where the surfaces are C^1 continuous. In the particular case where the tension parameters are all set to 1, the scheme reproduces a variant of the Catmull-Clark subdivision scheme. As an application, this scheme is used to generate surfaces of revolution from a given profile curve.

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Dual Contouring of Hermite Data

T. Ju, F. Losasso, S. Schaefer, J. Warren*

Keywords: *Implicit functions, contouring, crack prevention, quadratic error functions, polyhedral simplification.*

This paper describes a new method for contouring a signed grid whose edges are tagged by Hermite data (i.e. exact intersection points and normals). This method avoids the need to explicitly identify and process "features" as required in previous Hermite contouring methods. We extend this contouring method to the case of multi-signed functions and demonstrate how to model textured contours using multi-signed functions. Using a new, numerically stable representation for quadratic error functions, we develop an octree-based method for simplifying these contours and their textured regions. We next extend our contouring method to these simplified octrees. This new method imposes no constraints on the octree (such as being a restricted octree) and requires no "crack patching". We conclude with a simple test for preserving the topology of both the contour and its textured regions during simplification.

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Refined Error Estimates for Radial Basis Function Interpolation

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Keywords: *Scattered data Interpolation, positive definite functions.*

Interpolation by radial basis functions (RBFs) has become a well-established tool for use in reconstructing functions from scattered data in $\mathbb{R}^d$. More recently, similar functions, ones that are positive definite, have proved equally useful when reconstructing functions on tori and spheres.

Regardless of which manifold Ω ($\mathbb{R}^d$, T^d or S^d) is used, the pointwise error estimates take the form

$$|D^\alpha f(x) - D^\alpha s_{f,X}(x)|_{\infty,\Omega} \leq P^{(\alpha)}_{\Phi,X}(x)|f|_{\Phi,\Omega}, \qquad x \in \Omega,$$

where X represents the set of scattered data sites, $s_{f,X}$ is the interpolant to f and D^α is a differential operator. An accurate determination of the "power function" $P^{(\alpha)}_{\Phi,X}(x)$ has been central to determining the "rates of approximation" via interpolation. The bounds on the power function typically have been of the form $\mathcal{O}(h^r)$ where h represents the mesh norm of X and the power r depends on Φ. Far less attention has been paid to the constant $C^{(\alpha)}$ inherent in the $\mathcal{O}(h^r)$ term. However, estimates of $C^{(\alpha)}$ given previously were for theoretical purposes, and can often be so much larger than what is seen in numerical computations that they have little practical value. Our primary focus is to provide new, refined estimates for the constants $C^{(\alpha)}$; these turn out to be orders of magnitude smaller than previously known bounds, and also will apply to all the manifolds mentioned above.

The setting for these results are classes of reproducing kernel Hilbert spaces, which are also called *native spaces* in the RBF literature. However, one can "bootstrap" the *interpolation* error estimates for functions in the native space to get *approximation* error estimates of functions in general Sobolev spaces.

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Characterization of Semi-Hilbert Spaces with Application in Scattered Data Interpolation

T. Werther

Keywords: *Semi-Hilbert spaces, conditionally positive definite functions, locally convex vector spaces, interpolation, harmonic Hilbert spaces.*

Conditionally positive definite functions, in particular radial basis functions, and associated native Hilbert spaces have been extensively studied for the two cases where the native Hilbert space is a subspace of R^Ω, $\Omega \subset R^d$, and of $D'(R^d)$, cf. [3,6] and [4], respectively. We discuss the characterization of semi-Hilbert kernels and native Hilbert spaces in the more general context of locally convex

vector spaces for which the well known cases are special examples. The approach allows for various settings that are relevant in practice. It also includes examples of function spaces on Riemannian manifolds and on locally compact Abelian groups.

We further study the generalized optimal (minimal semi-norm) interpolation problem similar to [2,5] and state necessary conditions for unique solutions. This covers many interpolation problems encountered in practice, in particular, optimal recovery from generalized Hermite-Birkhoff data for which we give an example.

In those cases where the native Hilbert space is translation-invariant it enjoys immediate relations to harmonic Hilbert spaces [1]. As a consequence we can transfer sampling techniques from the theory of harmonic Hilbert spaces to this class of native Hilbert spaces.

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The Sylvester Resultant Matrix for Bernstein Polynomials

Joab R. Winkler*, Ronald N. Goldman

Keywords: *Sylvester resultant matrix, Bernstein polynomials.*

Resultants are used in several areas of applied mathematics, including robot motion planning, computer–aided geometric design, computer vision, and inverse kinematics for serial mechanisms. These applications have encouraged renewed interest in resultants, and many new theoretical results have been achieved. These theoretical results assume that the polynomials are expressed in the power basis [1], but the numerical superiority of the Bernstein basis with respect to the power basis requires that resultants for Bernstein polynomials be constructed directly if resultants are to be implemented in a floating point environment. This paper addresses this issue by considering the construction and properties of the Sylvester resultant matrix for Bernstein polynomials. It is shown that the combinatorial factors in the Bernstein basis functions destroy some of the symmetry properties of the Sylvester resultant matrix that hold for the power and scaled Bernstein bases. The matrices that transform the Sylvester resultant matrix between the power and Bernstein bases are derived, and an example is given.

The analysis of the computational robustness of the Sylvester resultant matrix in a floating point environment requires that its numerical condition be defined. It is shown that the condition number that may be used for the Bézout and companion resultant matrices [2,3,4] cannot be used for the Sylvester resultant matrix because it is not scale invariant, that is, the determinant can be made arbitrarily small or large by scaling the coefficients of one or both of the polynomials. This lack of invariance is due to the fundamental structure of the Sylvester resultant matrix – the coefficients of the polynomials are decoupled in this matrix, unlike the other resultant matrices.

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A Meshless Method for the Numerical Solution of PDEs by using Quasi–interpolation for Scattered Data

Zong Min Wu

Keywords: *Quasi–Interpolation, Scattered Data Approximation, Numerical Solution of PDE, Meshless Method.*

Meshless methods have become more and more popular recently, because the methods do not require having a partition of the domain or do not assume a structure of the knots. Thus the methods possess some advantage that save the time of need no computation of pre–treatment of the knots. However, most meshless methods still require solving a very large scaled linear system of equations. On the other hand, quasi–interpolation does not solve any linear system of equations at all to approximate the solution. A univariate example combining a meshless method and quasi–interpolation to solve ODEs can be found in [1]. Following the results of [2], where we have derived a quasi–interpolation scheme for scattered data, a new meshless scheme for the numerical solution of boundary value problems will presented in this talk. Some techniques in [3], [4], and [5] are used in the derivation.

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Error Estimates for Radial Basis Function Interpolation in Sobolev Spaces

Jungho Yoon

Keywords: *Radial basis function, interpolation, surface spline, 'shifted' surface spline.*

The accuracy of interpolation by a radial basis function ϕ is usually very satisfactory provided that the approximant f is reasonably smooth. However, for functions which have smoothness below a certain order associated with the basis function ϕ, no approximation power has yet been established. Hence, the purpose of this study is to discuss the L_p-approximation ($1 \leq p \leq \infty$) order of interpolation to functions in the Sobolev space $W_p^k(\Omega)$ with $k > \max(0, d/2 - d/p)$. We are particularly interested in using the 'shifted' surface spline, which actually includes the cases of the multiquadric and the surface spline. Moreover, we show that the accuracy of the interpolation method can be at least doubled when additional smoothness requirements and boundary conditions are met.

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Nonlinear Pyramid Transforms and Nonlinear Subdivision Schemes Based on Median-Interpolation: some Recent Results

Thomas P.-Y. Yu

Keywords: *Wavelet, subdivison schemes, nonlinear subdivision schemes.*

The speaker shall review a class of nonlinear wavelet-like transforms based on multiscale median-interpolating refinement, discusses their basic theoretical properties and their applications in robust denoising. Underlying these nonlinear pyramid transforms is a family of nonlinear subdivision schemes conceptually very similar to Deslauriers-Dubuc's scheme, except that one replaces linear Lagrange interpolation by nonlinear median interpolation. We shall discuss some of our recent results in analyzing the convergence and regularity of such nonlinear subdivison schemes.

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Geometric Interpolation by Cubic Polynomials

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Keywords: *Interpolation, geometric interpolation, parametric polynomial curve.*

It is well known that the interpolation of curve data by parametric polynomial curves essentially differs from the function case, i.e., this interpolation makes use of new ideas such as the geometric continuity, and the interpolation at unknown parameter values. As a rule, this geometric interpolation leads to nonlinear problems that are quite often difficult to deal with.

It has been conjectured that data based upon a smooth regular parametric curve in $\mathbb{R}^d$ can be geometrically interpolated at m points by polynomial parametric curve of degree $n = m - 1 - \lfloor (m-2)/d \rfloor$. So far, this conjecture has been confirmed in the asymptotic sense for a few particular choices of m and d. Also some general results in $d = 2$ were obtained, but there are no general results on geometric interpolation of arbitrary data for $d \geq 3$.

The case $n = d = 3$, and $m = 5$ will be studied in detail. A simple necessary and sufficient geometric condition concerning the data points only will be stated, and the uniqueness of the solution proved. Further, the extension of the approach to the general dimension d will be outlined.

At the end, some numerical results for $d = 3$ will be given.

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Lagrange Interpolation by Splines on Triangulated Quadrangulations

Frank Zeilfelder

Keywords: *Bivariate splines, triangulated quadrangulations, Lagrange interpolation, approximation order.*

There exists a vast literature on local Hermite interpolation by bivariate splines. On the other hand, since recently, no results of this type were known for Lagrange interpolation. Such schemes are advantageous in applications since they do not require derivatives at prescribed points. Jointly with Günther Nürnberger

and Larry L. Schumaker, we developed the first local Lagrange interpolation methods for C^1 splines on triangulations of arbitrary convex quadrangulations. The schemes are based on a fast coloring of the quadrilaterals with two colors. Therefore, the interpolating splines can be computed with linear complexity using the well-known BB-techniques. In addition, the interpolating splines yield optimal approximation order. We give numerical tests for smooth functions and real world data with up to 10^6 points which show the efficiency of the methods. In addition, I will talk about very recent extensions which are under development.

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Interpolatory Subdivision Schemes Generated by Splines

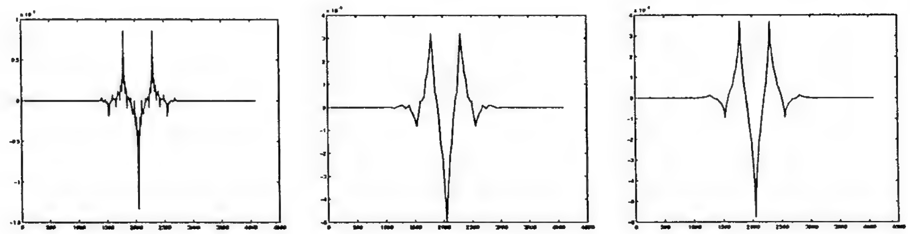
Valery A. Zheludev*, Amir Z. Averbuch

Keywords: *Subdivision, interpolatory spline, recursive filter.*

Interpolatory polynomial splines of even degrees constructed on uniform grid possess a property which is valuable for subdivision: In the midpoints between the points of interpolation these splines restore polynomials of the degree which is higher than their own degree. This property is called the *super-convergence* [2]. Basing upon this property, we introduce the following family of interpolatory subdivision schemes (ISS):

Given a function f^j defined on the grid $\mathbf{G^j} = \{k/2^j\}_{k=0}^{2^j-1}$: $f^j(k/2^j) = f_k^j$, we extend this function onto the grid $\mathbf{G^{j+1}}$ by the following rule. We construct the spline of an even degree $S_j^{2p}(x)$, which interpolates the function f^j on the grid $\mathbf{G^j}$: $S_j^{2p}(k/2^j) = f_k^j$. Then the samples f_k^{j+1} are defined as the values of the spline: $f_k^{j+1} = S_j^{2p}(k/2^{j+1})$, $k = 0, ..., 2^{j+1} - 1$. Due to the *super-convergence* property, the ISS which employs the splines of degree $2p$ restores polynomials of degree $2p+1$. It should be noted that, unlike traditional ISS [1], implementation of this scheme requires infinite impulse response (IIR) filtering of the array f^j. But, since transfer functions of the filters are rational, the operation the can be carried out via fast recursive filtering.

We provide more details about ISS based on quadratic spline which is most suitable for practical use. This ISS restores cubic polynomials. The sequence of sampled functions f^j converges to a function f belonging to $C^2[0,1]$. As for implementation, the passage from f^j to f^{j+1} is conducted as follows: $f_{2k}^{j+1} = f^j(k)$, $f_{2k+1}^{j+1} = s^j(k)$, where values $s^j(k)$ are derived from f^j by the cascade of elementary recursive filters: $x(k) = 4\alpha(f^j(k) + f^j(k+1))$, $x_1(k) = x(k) - \alpha x_1(k-1)$, $s^j(k) = x_1(k) - \alpha s^j(k+1)$.The parameter $\alpha = 3 - 2\sqrt{2} \approx 0.172$. Computational cost of computation of a value f_{2k+1}^{j+1} is 3 M(ultiplications) and 3 A(dditions). For comparison, the commonly used 4-point ISS, where samples f_{2k+1}^{j+1} are derived from the values of cubic Lagrange polynomials, requires 2M +3A, but the regularity of the limit function is inferior to the regularity of the limit function of the above scheme. The 6-point ISS based on quintic polynomials, which produces the limit function of approximately the same regularity as the spline ISS, requires 3M+5A. Thus, we argue that ISS based on quadratic splines is competitive with conventional ISS based on polynomial interpolation. In the figure we display second derivatives of the fundamental functions of the 4-point polynomial ISS (left picture), of the 6-point polynomial ISS (central picture) and of the ISS based on quadratic splines(right picture).



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Interpolatory Vector Subdivision Schemes

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Keywords: *Vector subdivision, interpolation.*

In (scalar-valued) stationary subdivision, the concept of interpolation is fairly immediate: we want the values at each step to be preserved in the next step, *i.e.*, we perform an upsampling and then calculate new, additional values for the gaps (*interpolatory subdivision*). Consequently, in a convergent subdivision scheme, we find the initial data as values of the limit function at the integers.

This concept does not extend immediately to the vector case, since there, at each point, the limit function (under standard assumptions) is a multiple of a fixed vector [1]. Therefore we can not expect the initial vector data to reappear as (vector) function values in the limit. One way out of this is Hermite

interpolation [2, 3], where the various components of the initial data vectors show up as derivatives of the limit function.

We want to keep interpreting the initial data as values of the limit function at certain points, though. To this end, we introduce the concepts of data preservation and quasi-data preservation and show how these lead to interpolating vector subdivision schemes.

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Solving Boundary Integral Equations on Subdivision Surfaces

D. Zorin

Keywords: *Boundary integral equations, subdivision surfaces, Stokes equation, Navier equations, linear elasticity, Galerkin and Nyström methods, quadrature rules.*

Subdivision is most commonly used in computer graphics and CAGD to construct smooth or piecewise smooth surfaces. Recently it was shown that bases constructed using subdivision are useful for numerical simulation, solution of shell equations in particular. In this talk, we further explore application of subdivision surfaces in numerical simulation.

Many 3D problems such as the Stokes equation or Navier equations of linear elasticity can be reduced to 2D problems defined on the boundary using potential theory. Such methods are particularly valuable when the solution of the 3D problem (e.g. displacements in the case of elasticity) needs to be evaluated only on the boundary. Subdivision surfaces are a convenient representation for the boundary as it is a parametric representation, which makes it possible to treat the functions defined on the surface and the surface itself in the same way. At the same time subdivision can be used to represent surfaces with arbitrary topology and sharp features.

The 2D equations on surfaces that we consider are integral equations with possibly singular kernels. Approaches to solving such systems include Galerkin and Nyström methods and require quadrature rules to compute coefficients of the discretized systems.

We discuss quadrature rules for integrating functions on subdivision surfaces, including quadrature rules for singular functions arising in Stokes and elasticity equations, and consider applications of such rules.

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